VU Rendering 2012

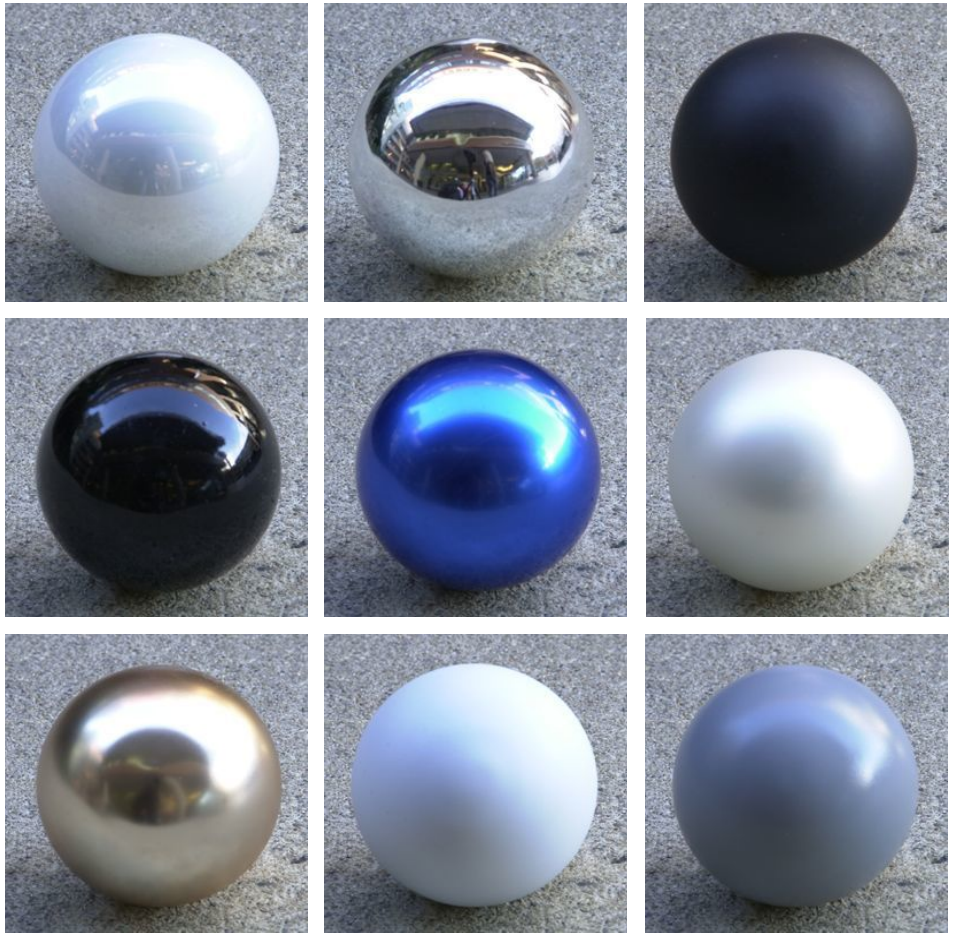
Unit 3: Surface Models

What gives a material its appearance?

© Jan Jinda

Surface Reflectance

- These spheres look different because they have different surface reflectance properties



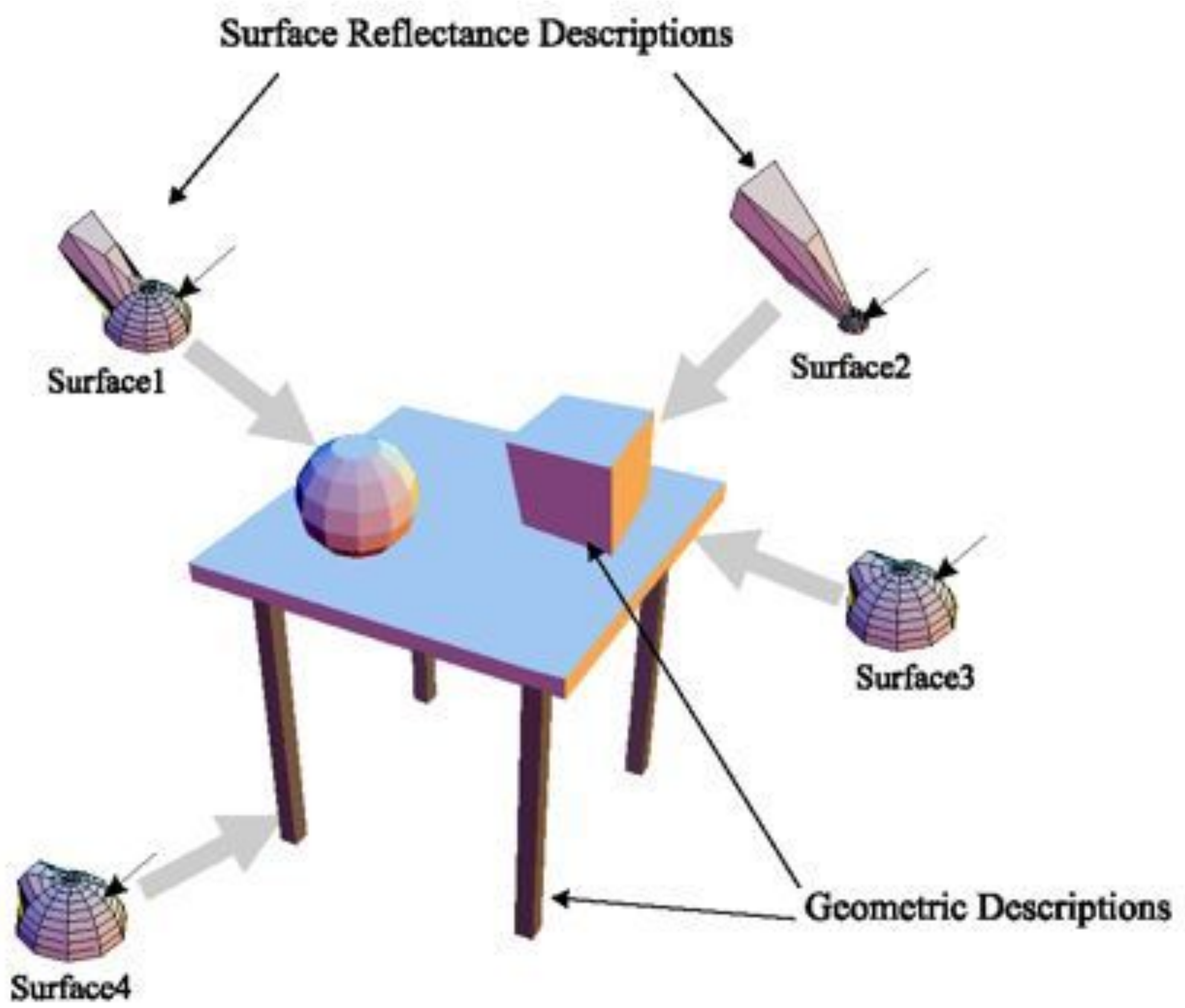
Surface - Light Interaction

- Key term of the equation for today:

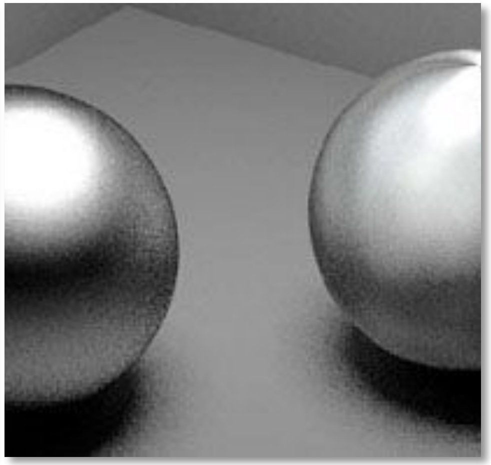
$$\rho(x, x', x'')$$

- Encodes how light from a given direction is modified upon reflection from a surface
- Has to be answereable for all directions and surface points in a scene

Surfaces in a Scene



Definition: BRDF

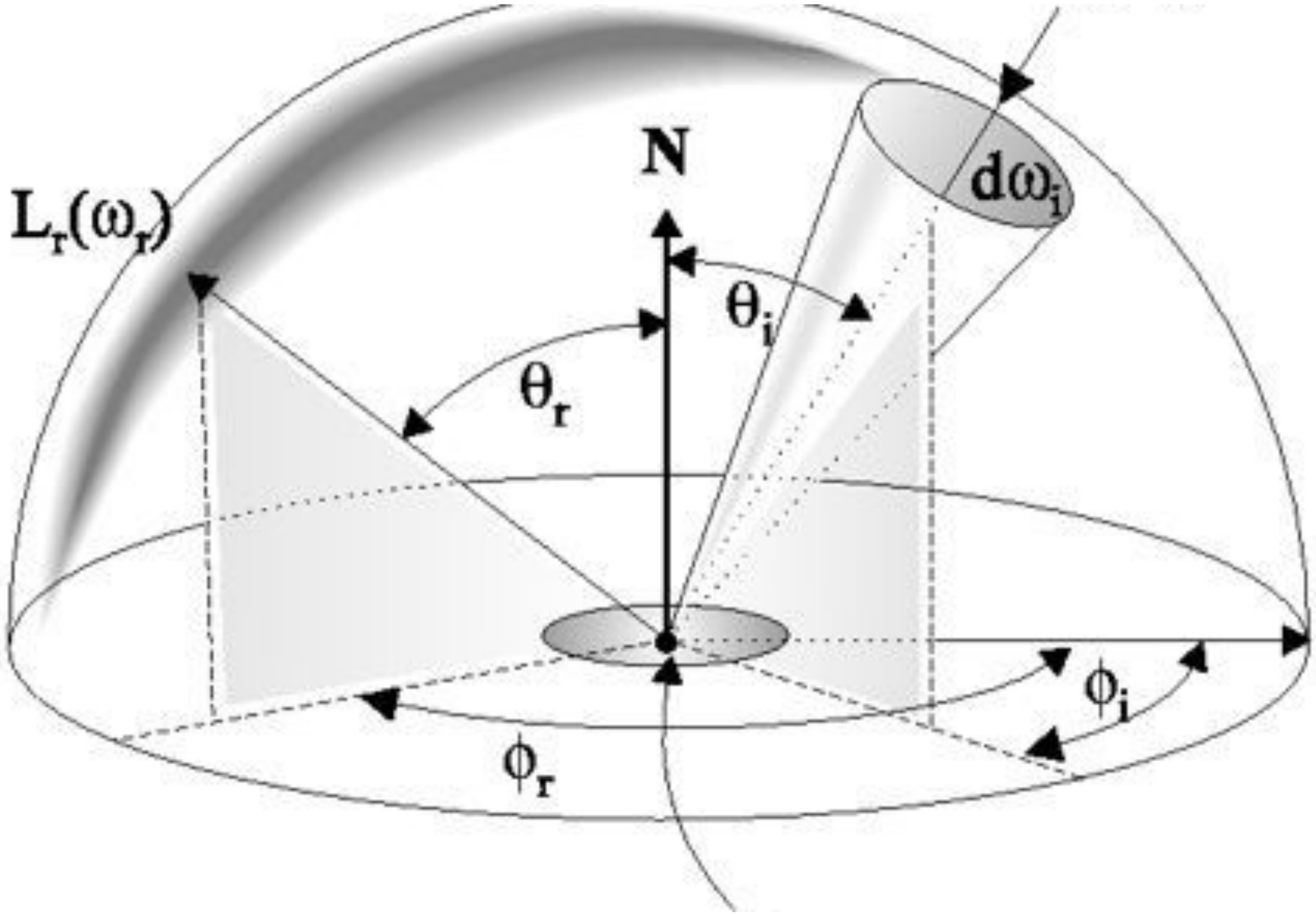




- Bidirectional Reflectance Distribution Function

$$f_r(\omega_{\mathbf{i}} \rightarrow \omega_{\mathbf{r}}) \equiv \frac{L_r(\omega_{\mathbf{r}})}{L_i(\omega_{\mathbf{i}}) \cos\theta_i d\omega_i}$$

- Hemispherical function
- 6 - dimensional (location, 4 angles, wavelength)
- Unit: sr -1
- Determines fraction of incident energy scattered in each direction

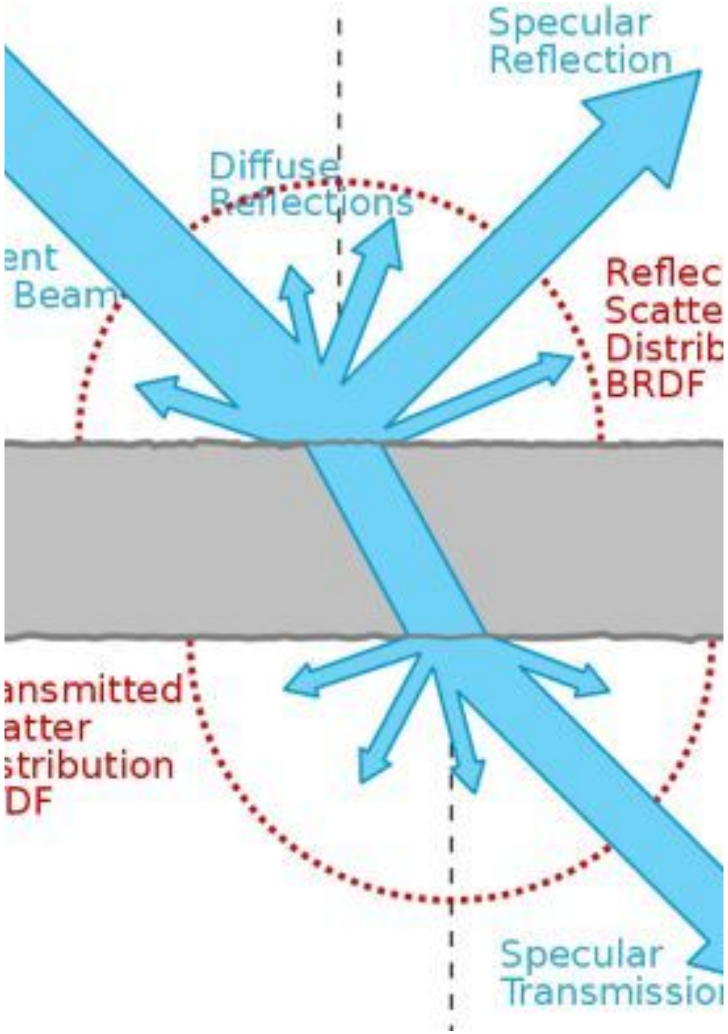
BRDF Geometry



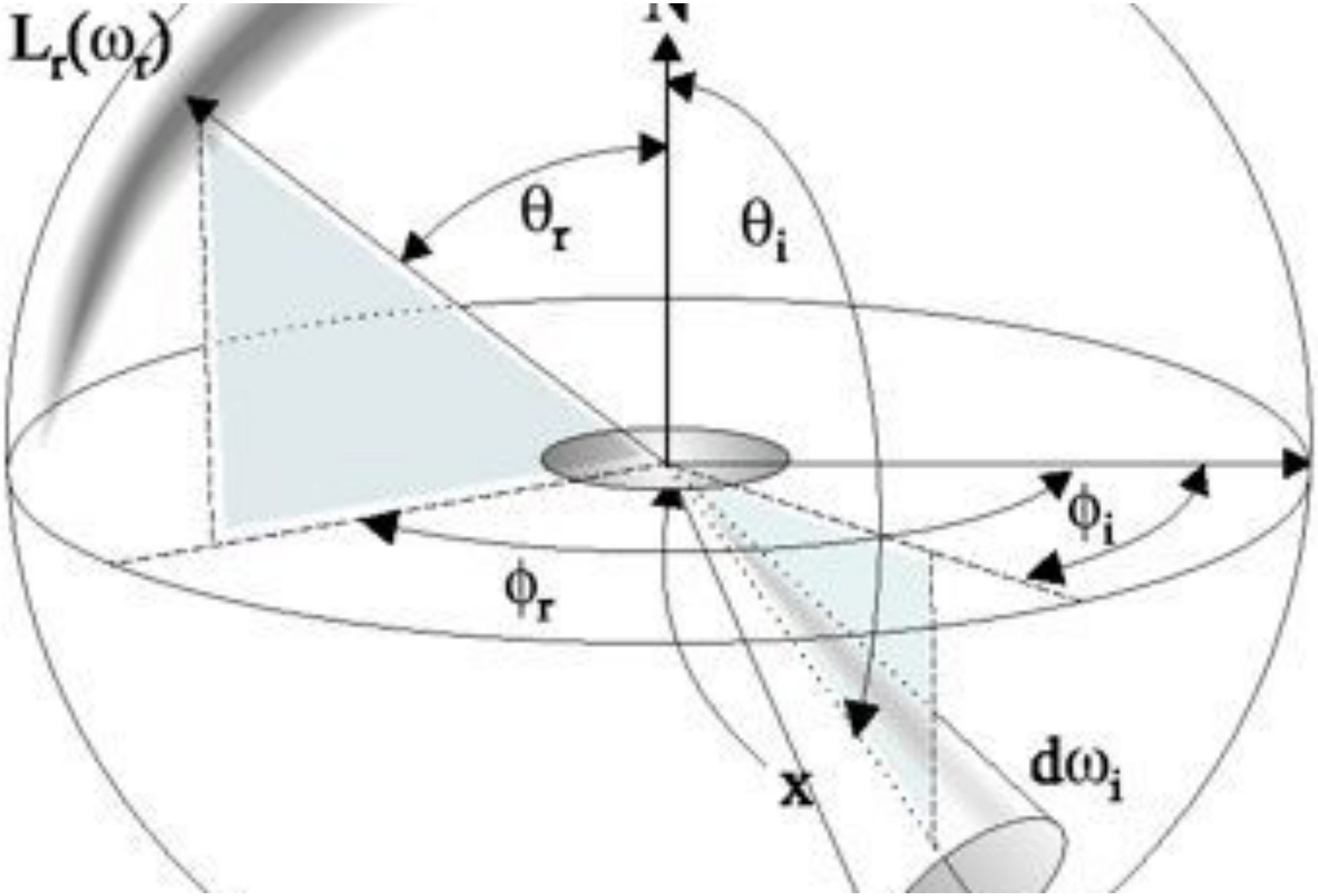
BTDF, BSDF

- **BTDF**: Bi-directional Transmission Distribution Function
- Similar to BRDF, but for rays that point into the material

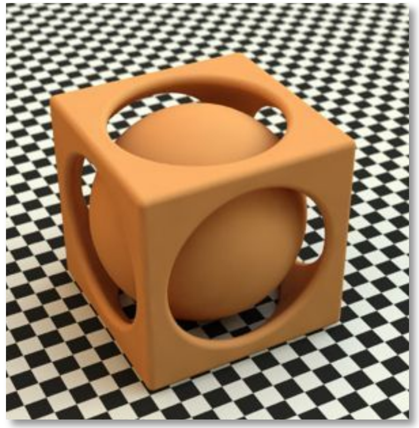
- Combination of BTDF & BRDF: **BSDF**
- Bi-directional Scattering Distribution Function
- Defined over entire sphere
- Rarely used

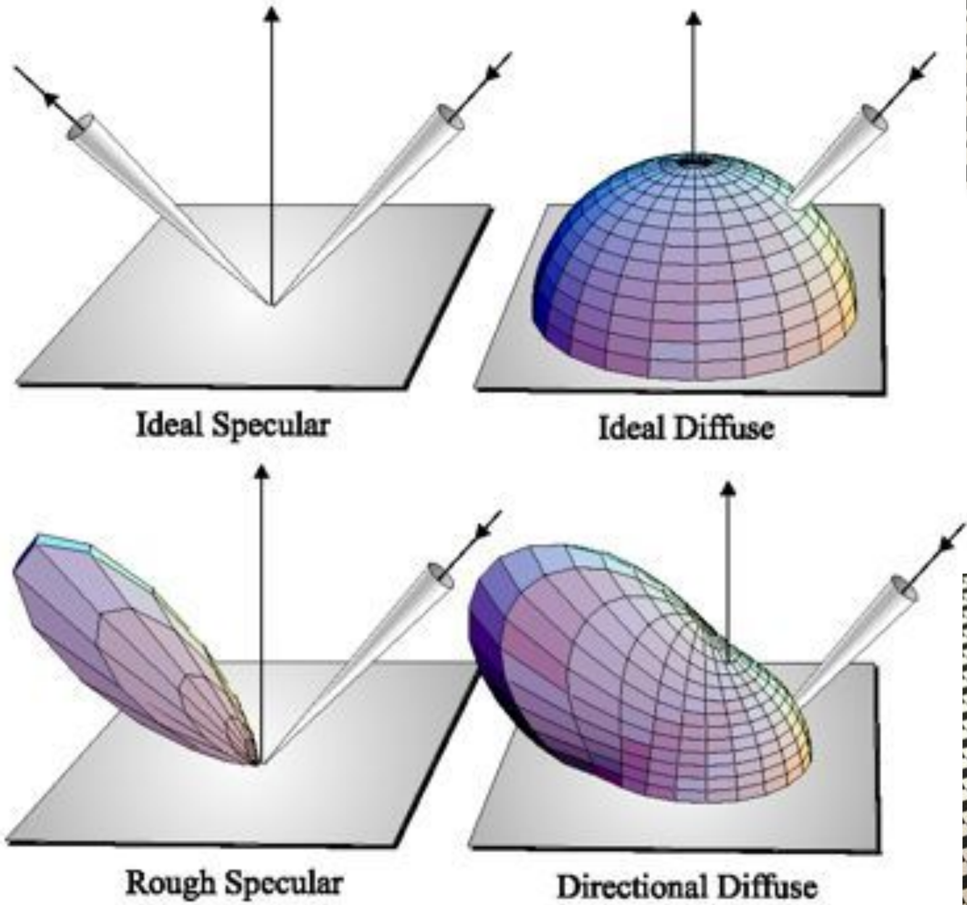


BSDF Geometry

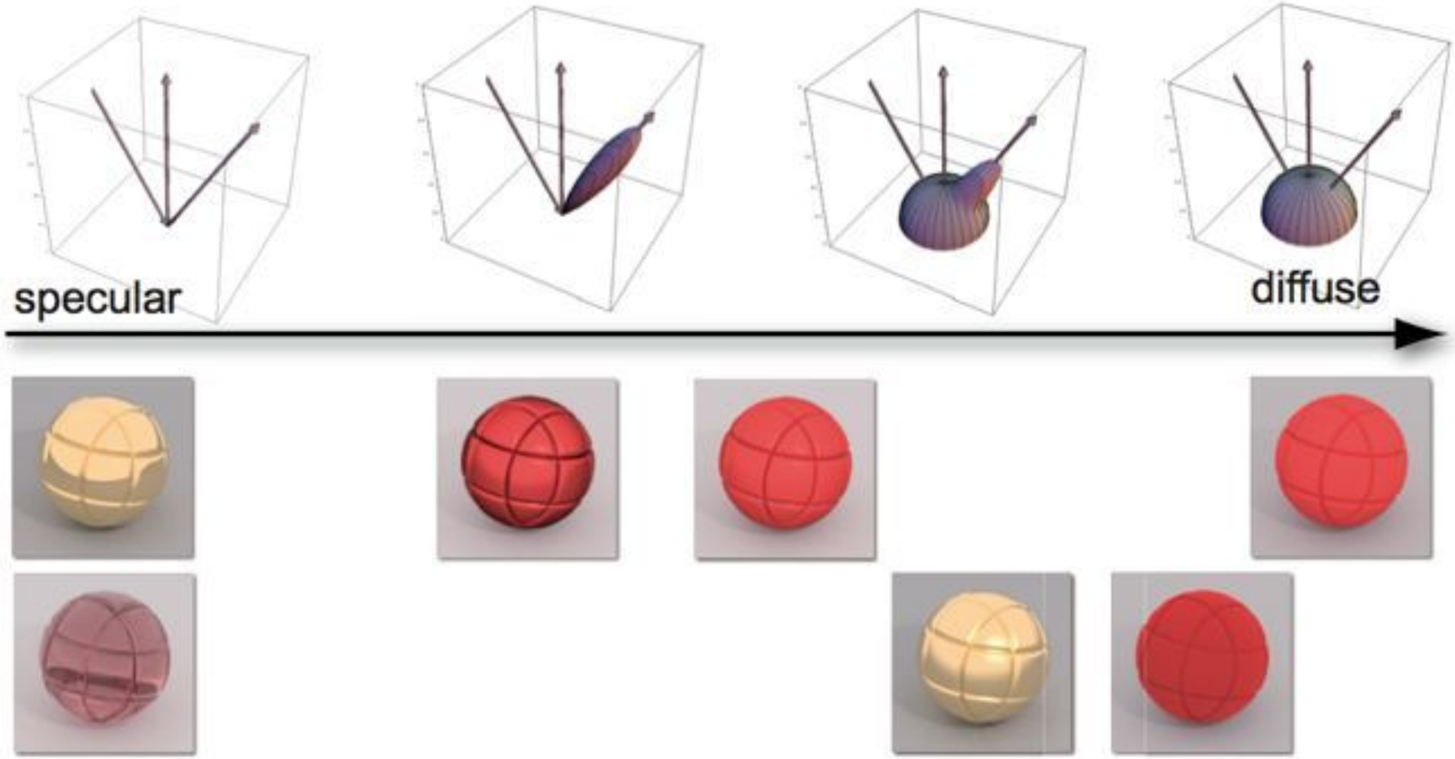


BRDF Specimens





Reflection Types



BRDF: Isotropy vs. Anisotropy

- Isotropy
  - Rotational invariance
  - Holds for a large number of surfaces
  - Reduces the number of variables by one
  - No alignment needed

$$f_r(\theta_i, \phi_i + \Delta\phi, \theta_r, \phi_r + \Delta\phi) = f_r(\theta_i, \phi_i, \theta_r, \phi_r)$$

- Anisotropy
  - Reflectance properties exhibit change with respect to rotation of the surface around the normal vector

Example Isotropic vs. Anisotropic

BRDF Requirements

- BRDF representations ought to:
  - use reasonable amounts of storage
  - faithfully capture the key features of the reflection characteristics
  - permit fast and easy sampling by Monte Carlo methods
    - Apart from perfectly diffuse surfaces and perfect mirrors, reflection properties are basically only tractable through MC rendering
    - Possibility of casting of rays according to distribution function is crucial

BRDF Data Sources

- Two fundamental approaches to the problem of representing BRDFs exist:
  - Explicit storage of tabulated measurements or simulation results
  - Approximation through analytical functions

BRDF Datasets

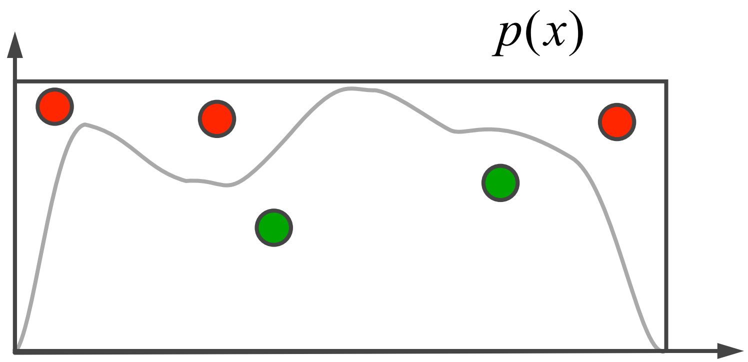
- Very memory - intensive when naively stored as full set of finely spaced samples (16 x 90 x 360 x 90 x 360 ...)
- Compression essential
- Hard and time-consuming to measure accurately
- Bad stochastic sampling characteristics (rejection sampling)
- Necessary for verification purposes

Gonioreflectometer @ NIST

- Expensive, hard to maintain and operate
- Generates huge amounts of data

Sampling of Measured BRDFs

- Bad stochastic sampling characteristics
  - Analysing data in advance or
  - Rejection sampling
    - Propose a sample, accept it if it passes a certain test
    - Efficiency depends on the acceptance rate



BTF

- Bi-directional Texture Function
- Similar to BRDF, only for entire textures (carpets, wood grain, cloth)
- Adds positional information to combine individual BRDF datasets
- Active research area with several problematic areas
  - Acquisition of samples
  - Generation of seamless textures from BTF data

BTF Examples

Requirements for Analytical BRDFs

- **Reciprocity**
  - Sampling directions can be interchanged
  - Due to Helmholtz reciprocity principle – a fundamental law of physics

$$f_r(\omega_{\mathbf{i}} \rightarrow \omega_{\mathbf{r}}) = f_r(\omega_{\mathbf{r}} \rightarrow \omega_{\mathbf{i}})$$

- **Energy conservation**
- Fast evaluation
- Expressivity
- Easy stochastic sampling

Analytical BRDFs

- Empirical models
  - Lambert, Phong, Blinn, Lafortune, …
  - Superposition of different components
  - (No physical background)

- Physically based models
  - Torrance-Sparrow, Cook-Torrance, Kajiya, He-Sillion-Torrance-Greenberg (HTSG), …
  - Physical material constants needed

Empirical Models

- Lambert: only diffuse component
- Phong: generalized cosine lobe
- Ward: anisotropic

- Can be combined for higher realism
- Energy conservation dependent on coefficients and combination (esp. for Phong)
- Easy to sample
- Generalizations possible

Perfectly Diffuse

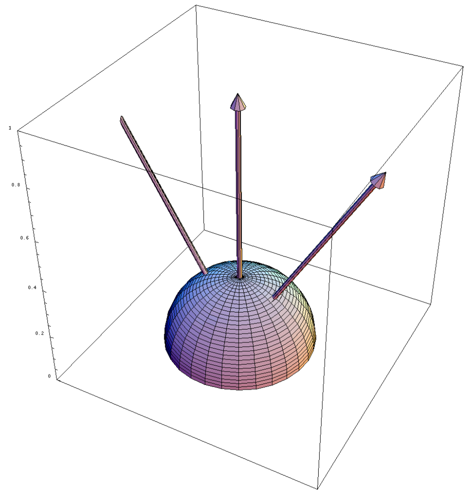
- Reflect the incoming light equally in all directions over the hemisphere
- Viewing direction independent
- E.g.
  - Lambert
  - Oren-Nayar
  - …

Lambert Surface

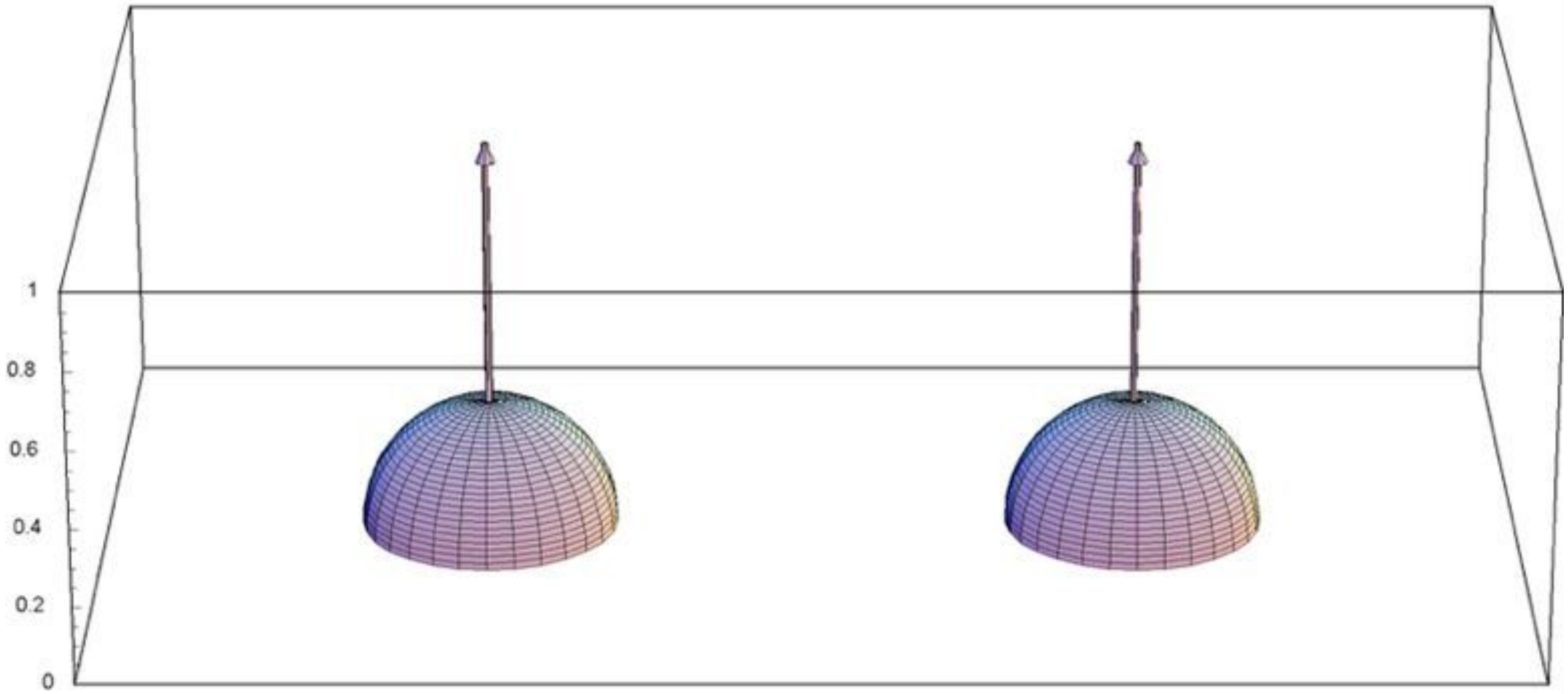
- Perfectly diffuse surface
- Light that leaves a surface is proportional to the cosine of the incident angle
- Colour defined by wavelength-dependent diffuse absorption coefficient k_d

$$f_r(\lambda) = \frac{1}{\pi} \cdot k_d(\lambda)$$

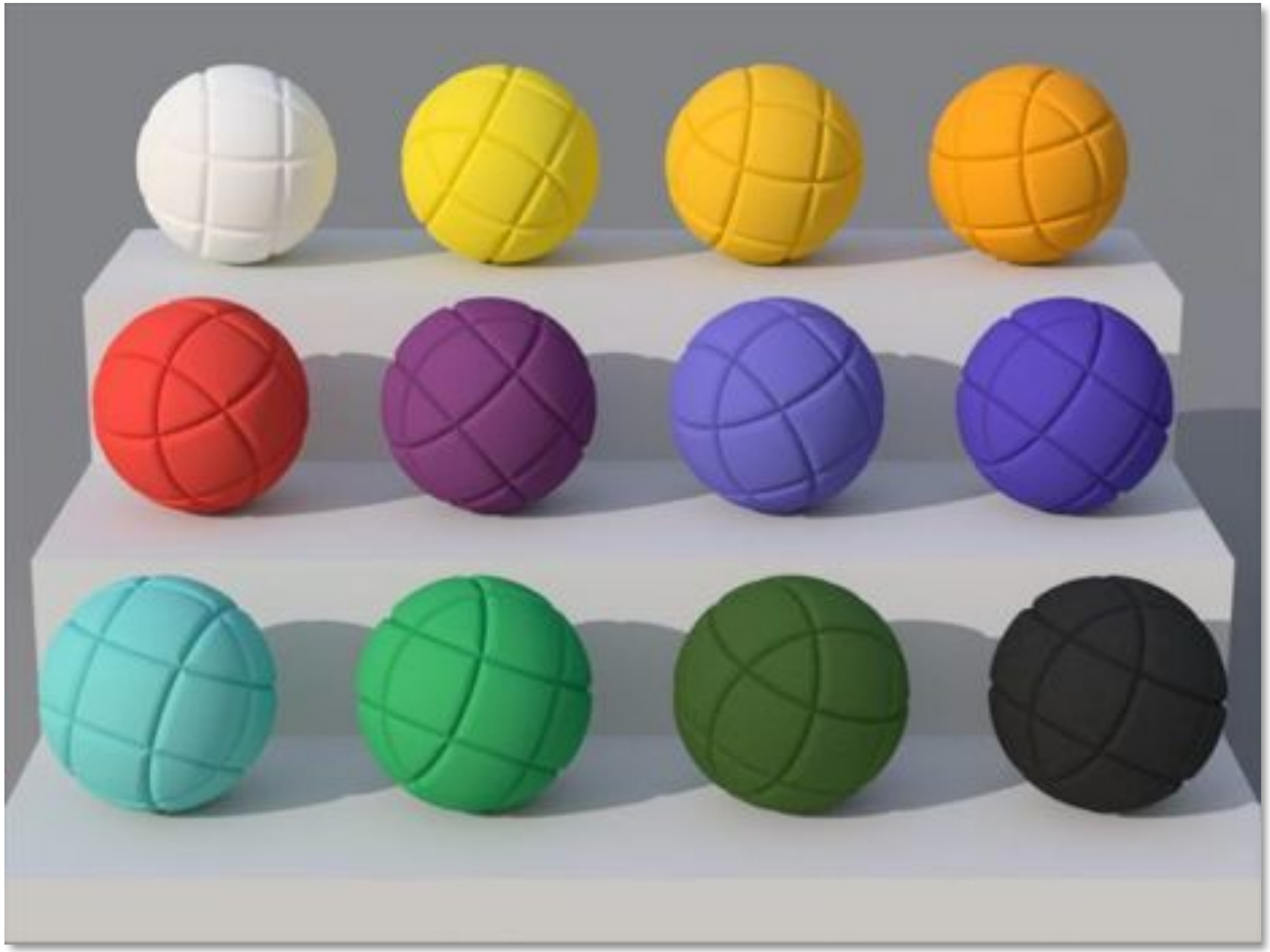
$$pdf(\theta_i, \lambda) = \frac{\cos\theta_i}{\pi} \cdot k_d(\lambda)$$



Lambert BRDF Shape & Size



Lambert Surfaces



Oren-Nayar Surface

- Microfacet-based diffuse surface
- Identical to Lambert's cosine model for sigma = 0
- More and more retro-reflective for increasing sigma

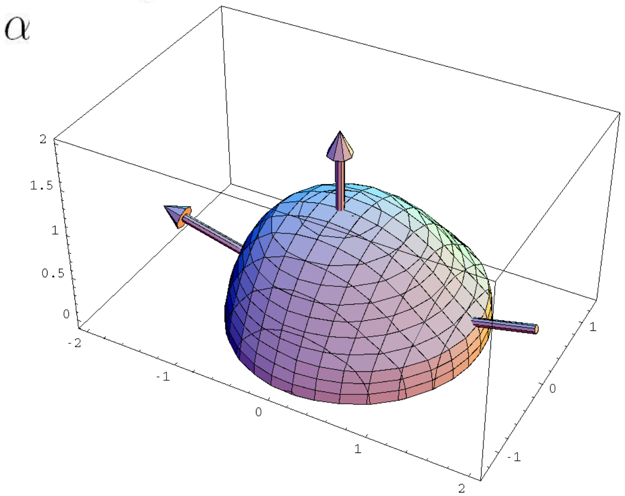
$$f_r(\omega_i, \omega_o) = \frac{\rho}{\pi}(A + B \max(0, \cos(\phi_i - \phi_o)) \sin \alpha$$

$$A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)}$$

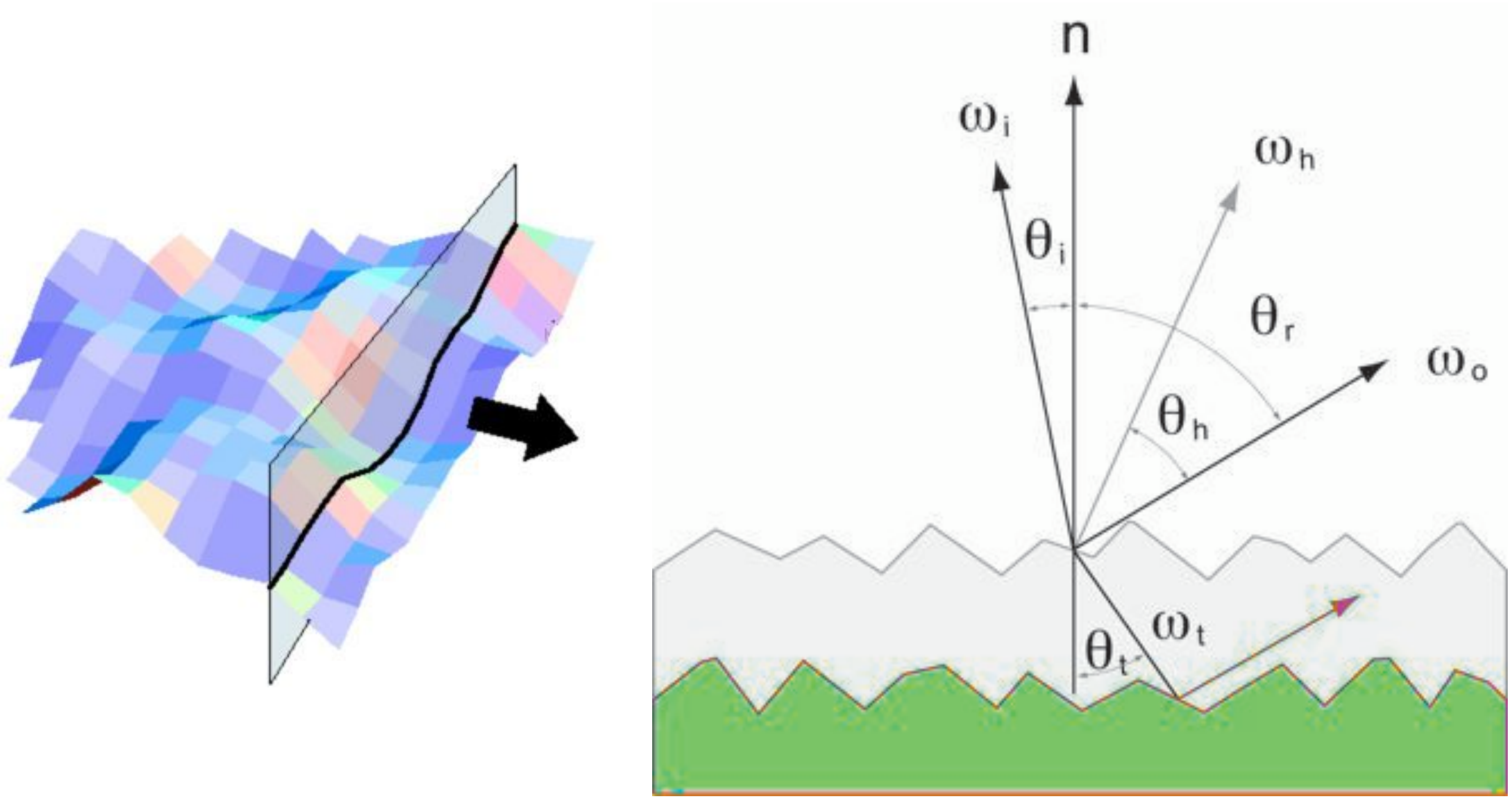
$$B = \frac{0.45\sigma^2}{\sigma^2 + 0.09}$$

$$\alpha = \max(\theta_i, \theta_o)$$

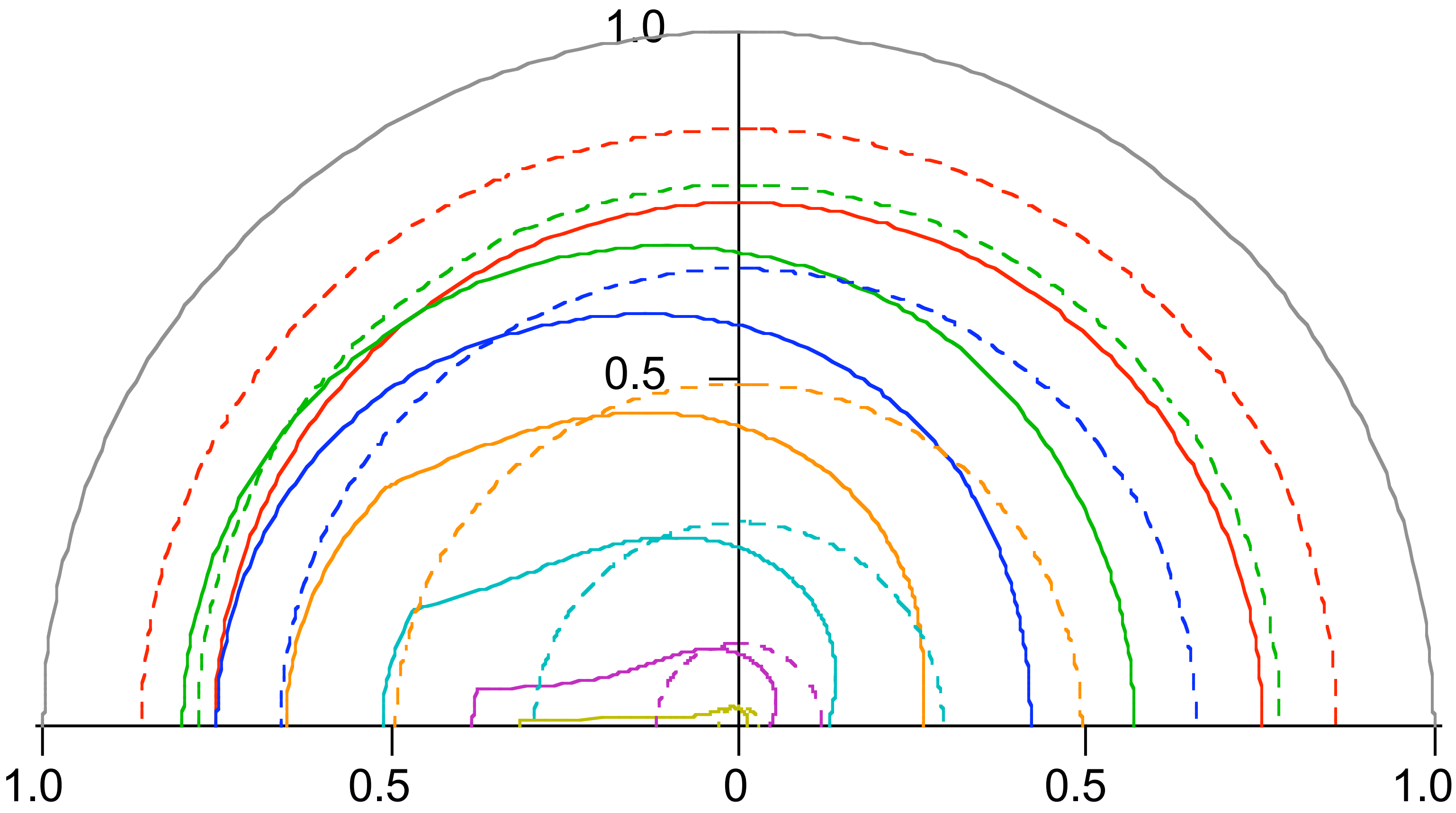
$$\beta = \min(\theta_i, \theta_o)$$



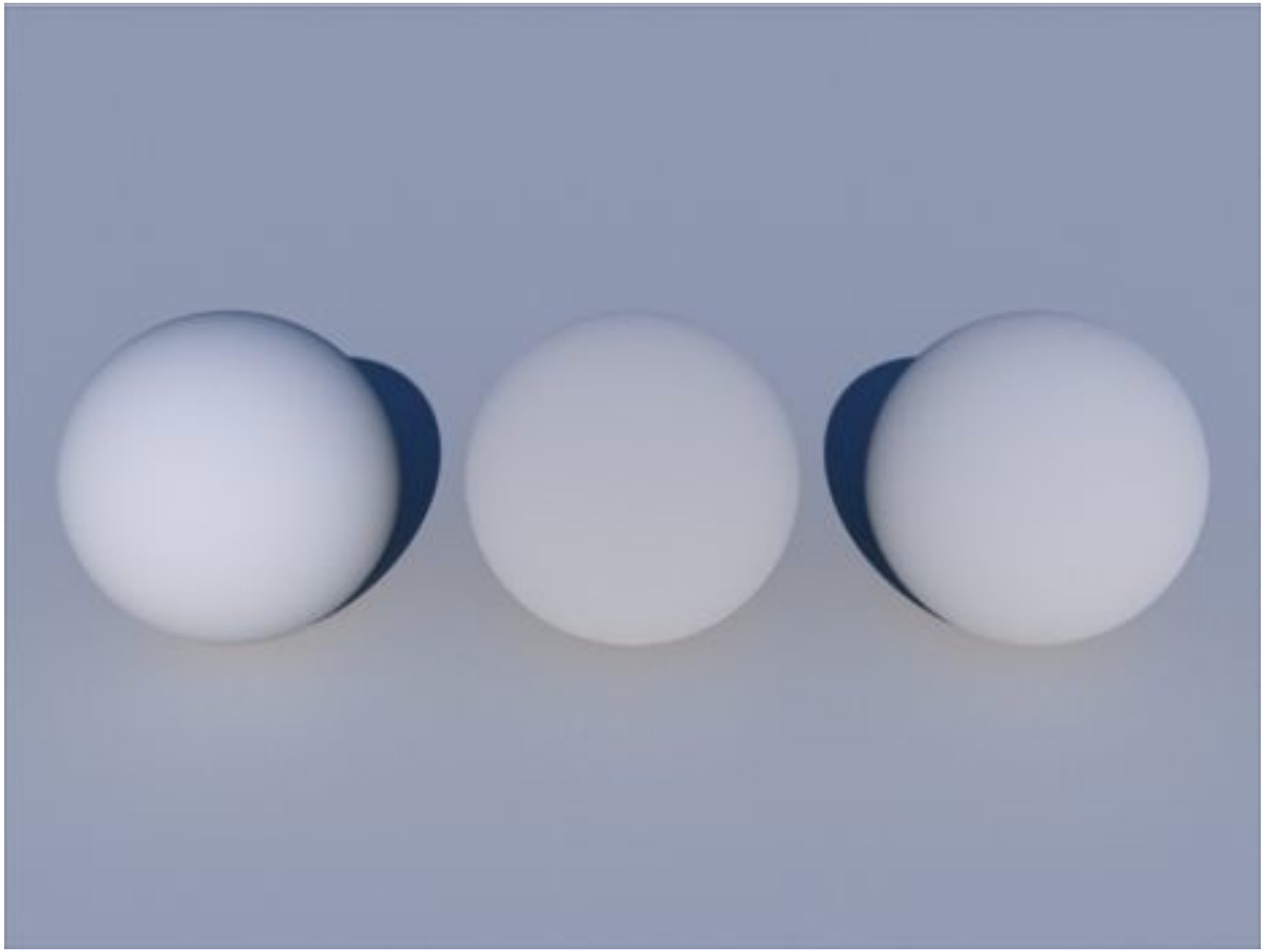
Microfacet Surfacs



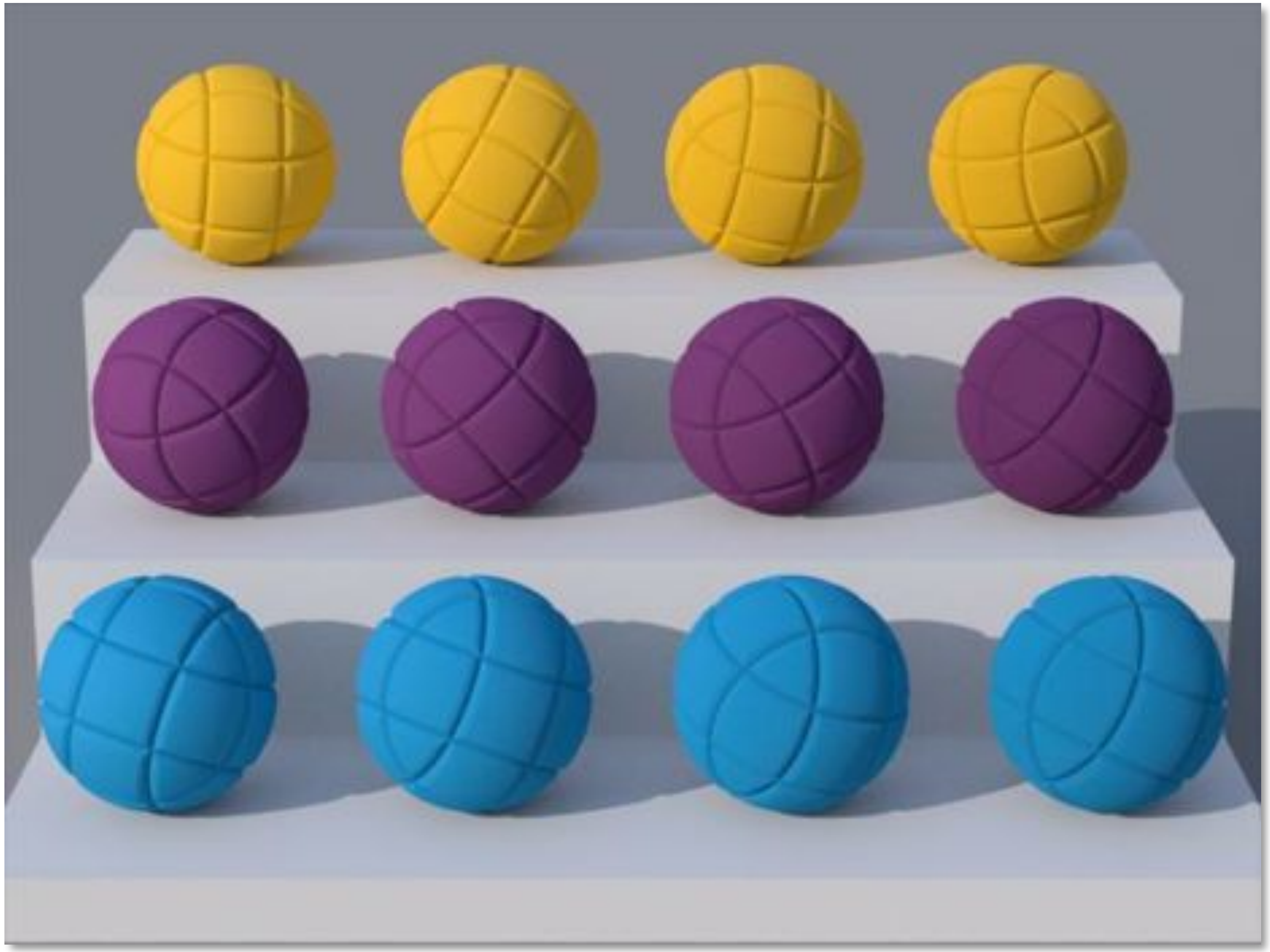
Medium vs. Rough Oren-Nayar



Oren-Nayar Example



Oren-Nayar Surface Example



Rough Specular

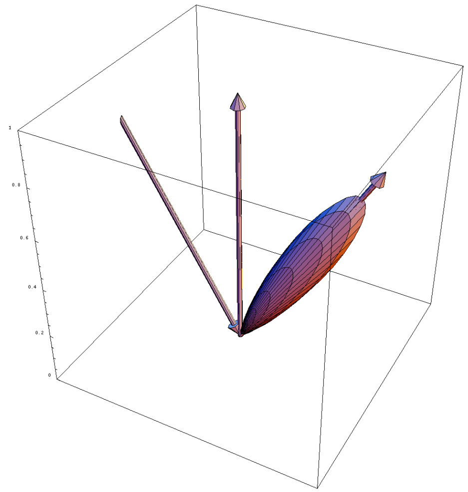
- Reflect light not only in the ideal direction
- „Highlight“
- Some of the light is reflected slightly off from the ideal specular angle.
- E.g. Phong: Size of the highlight can be changed with exponent

Phong Surface

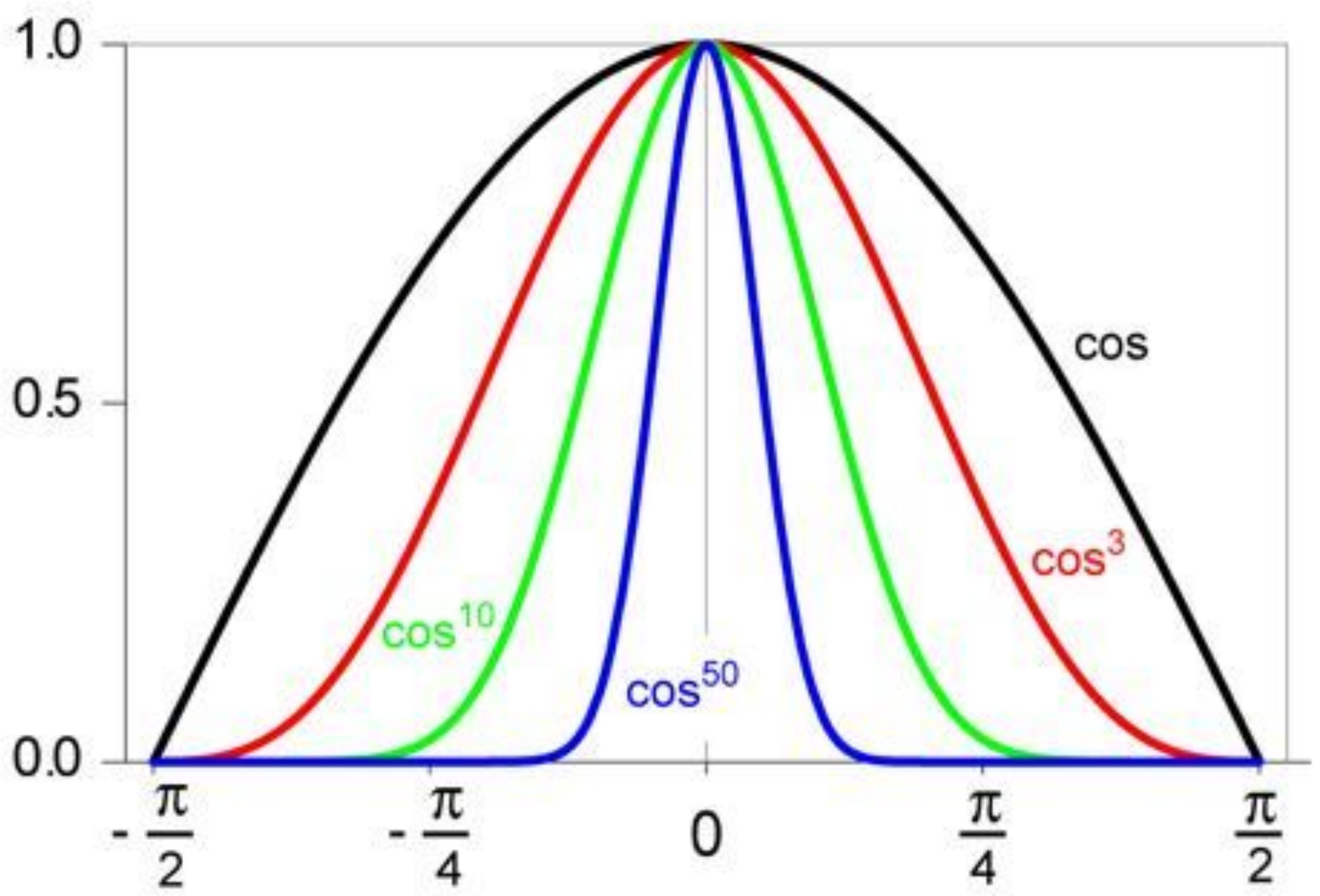
– Adds specular highlight

$$f_r(\omega_i, \omega_o) = \frac{\rho_d}{\pi} + \rho_s \frac{n+2}{2\pi} \cos(\omega_i \cdot R(\omega_o, \mathbf{n}))^n$$

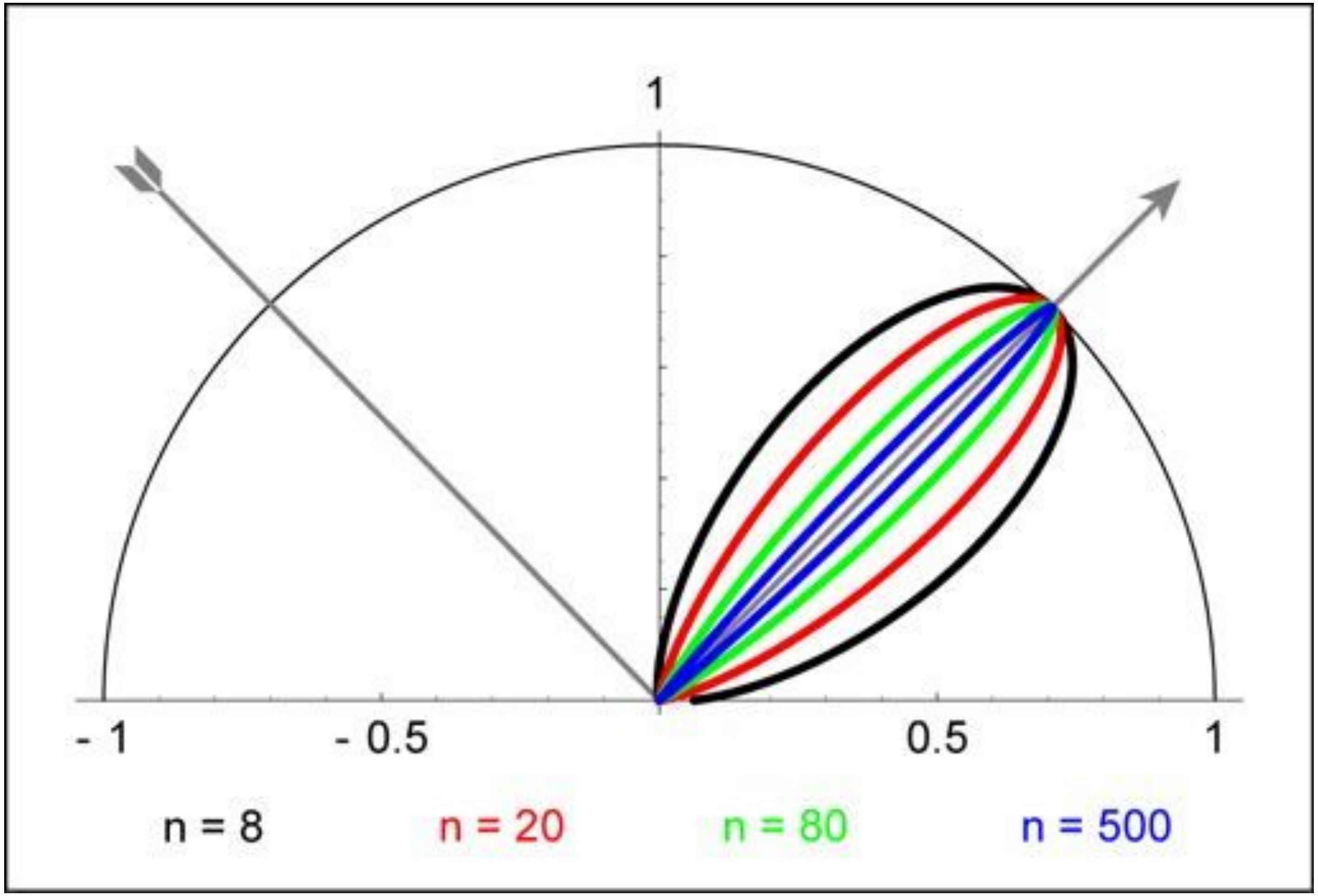
$$pdf(\omega_i, \omega_o) = \frac{n+1}{2\pi} \cos(\omega_i \cdot R(\omega_o, \mathbf{n}))^n$$



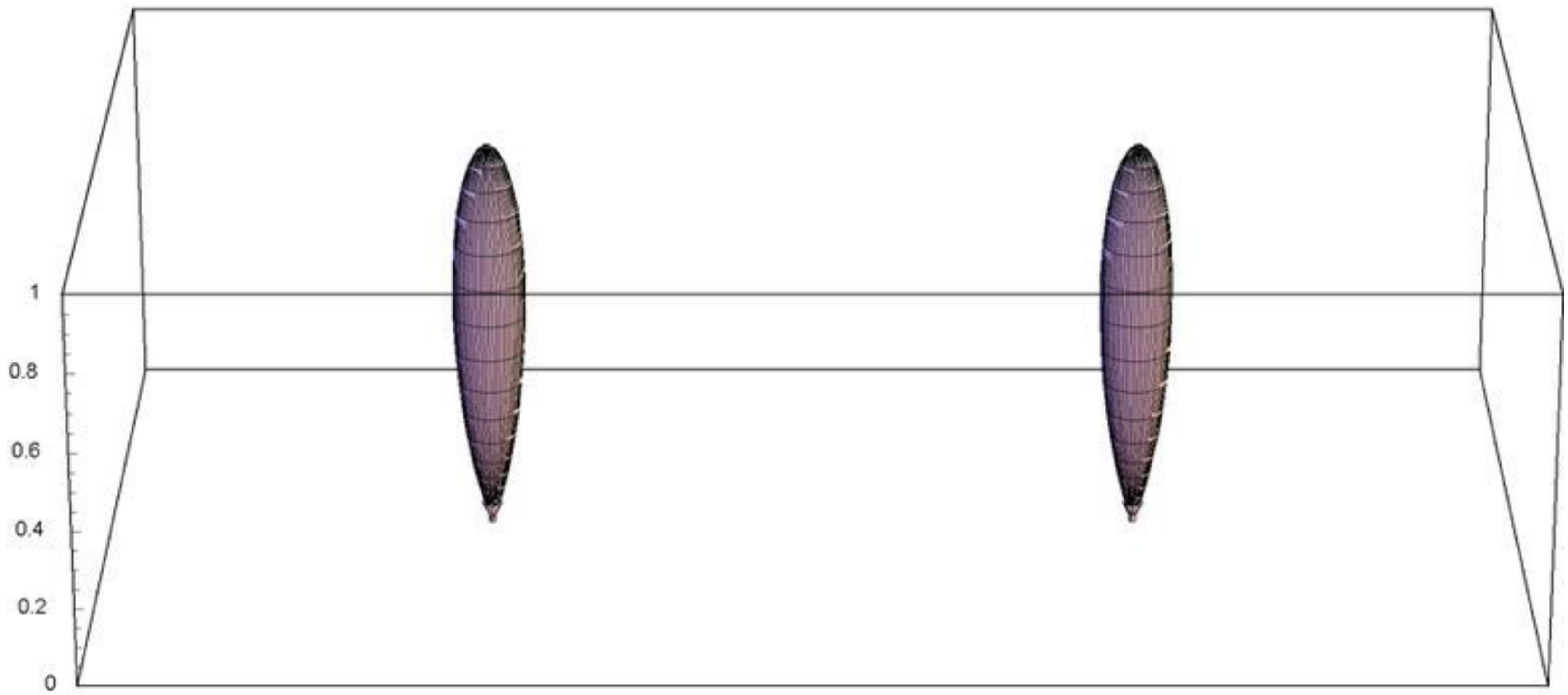
Cosine to the Nth



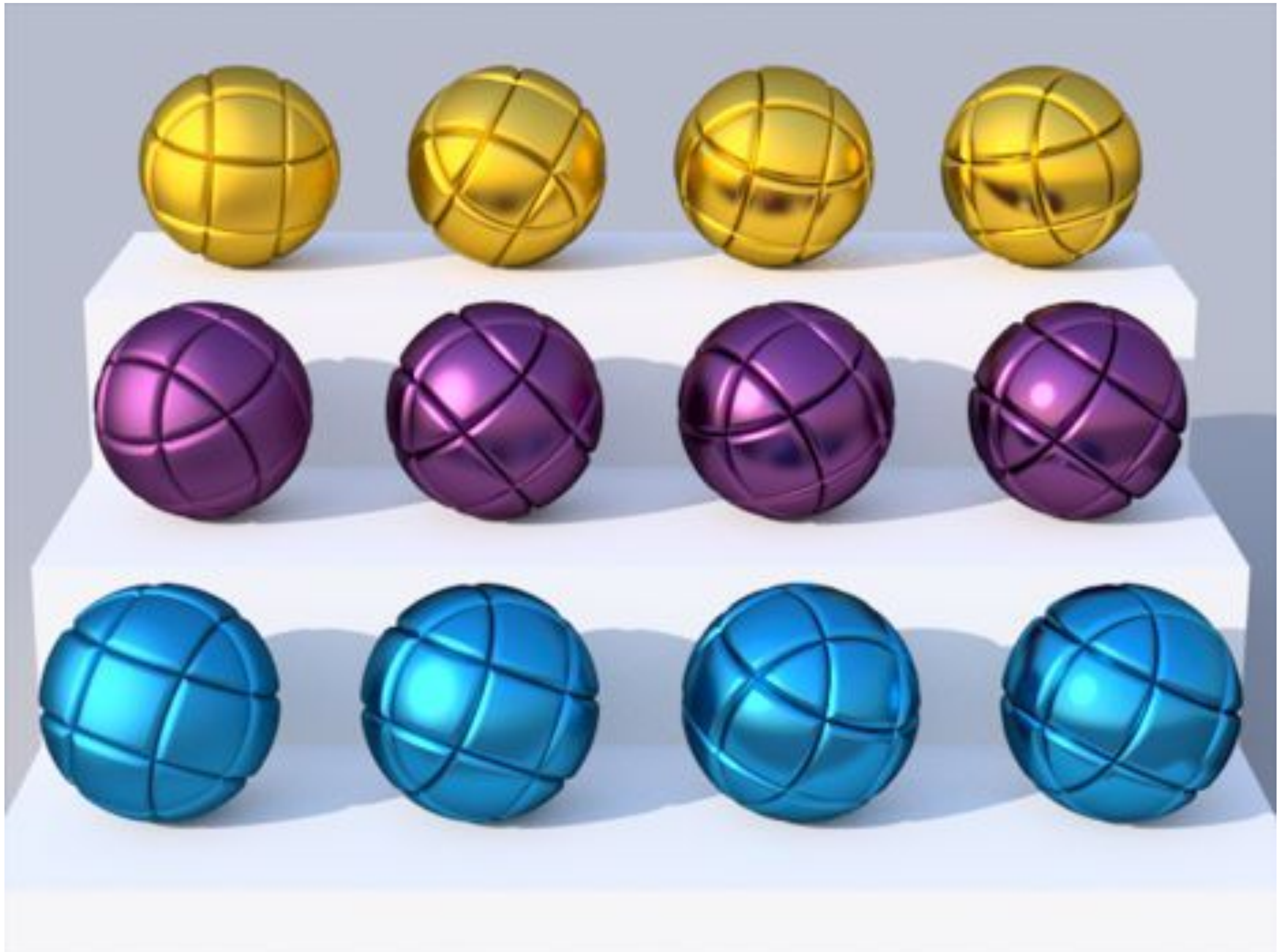
Cosine



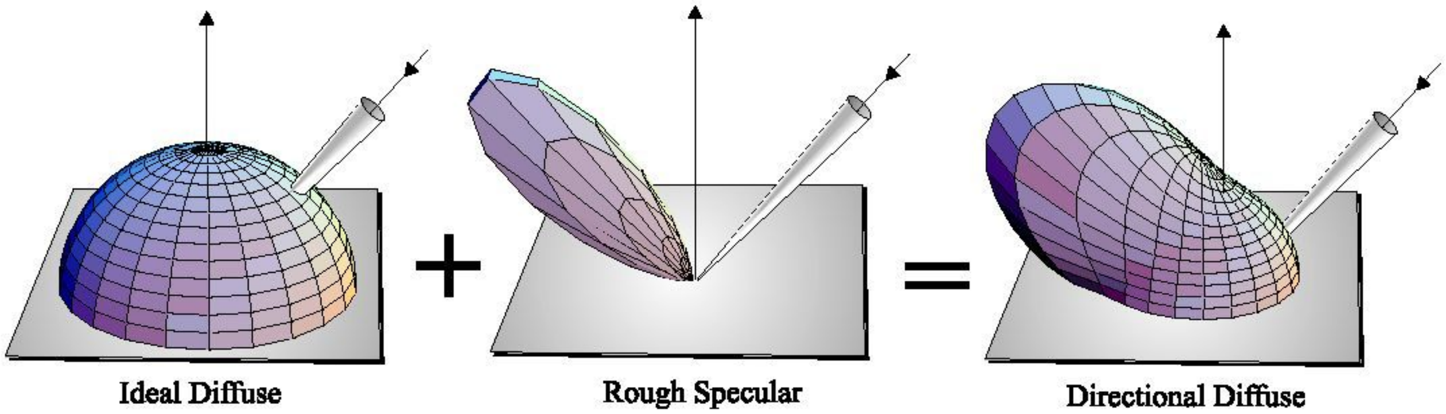
Plain Phong Lobe



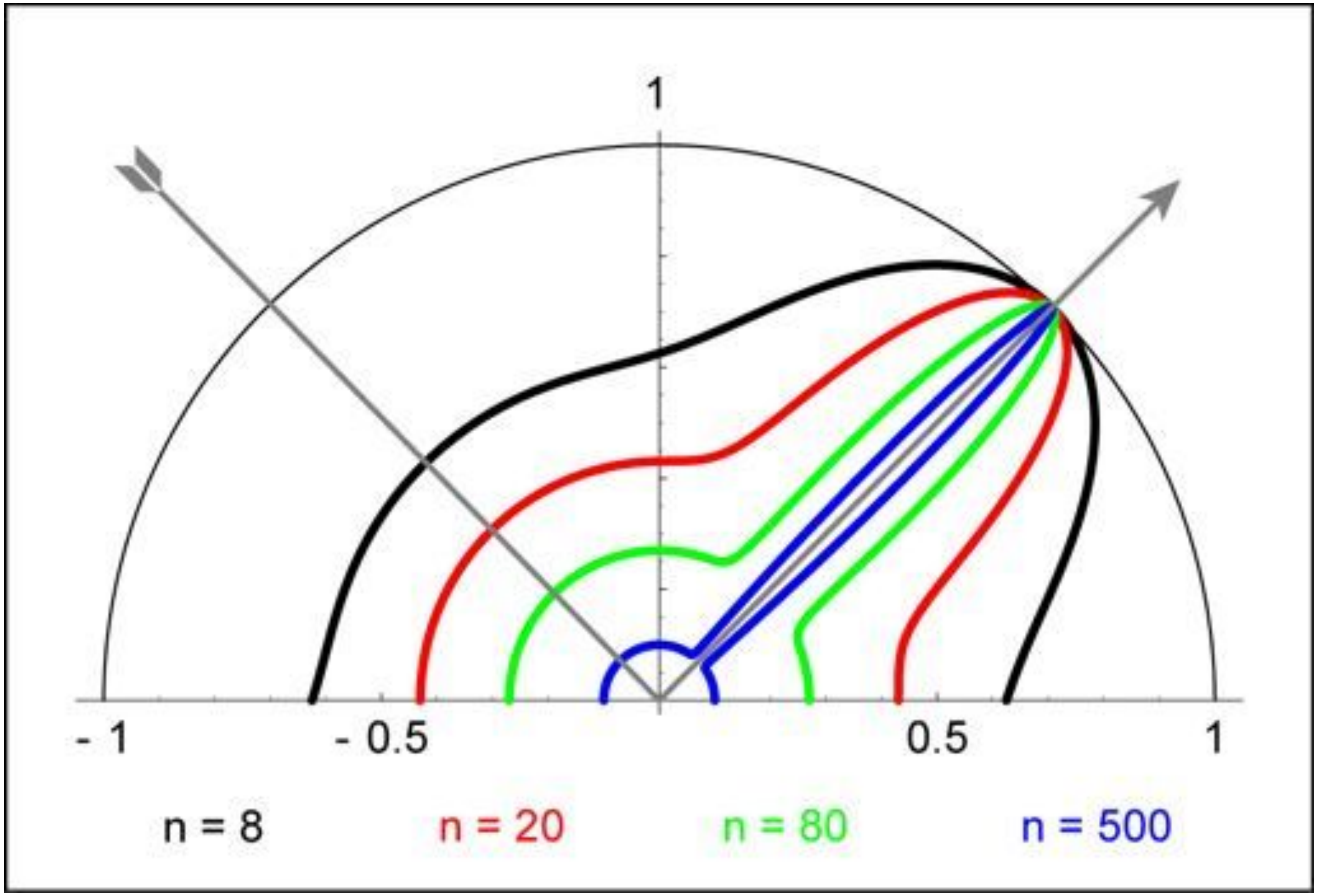
Plain Phong



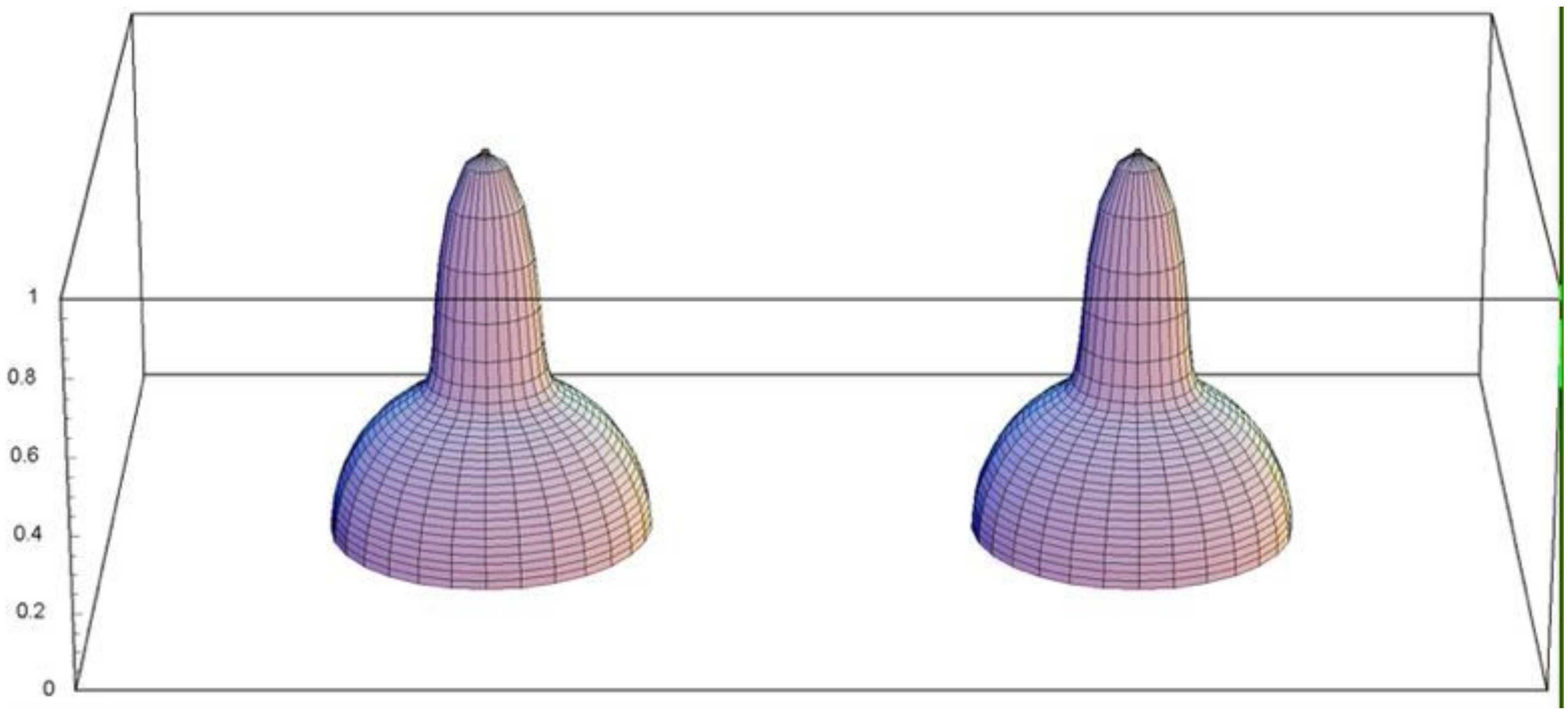
Superposition of BRDFs



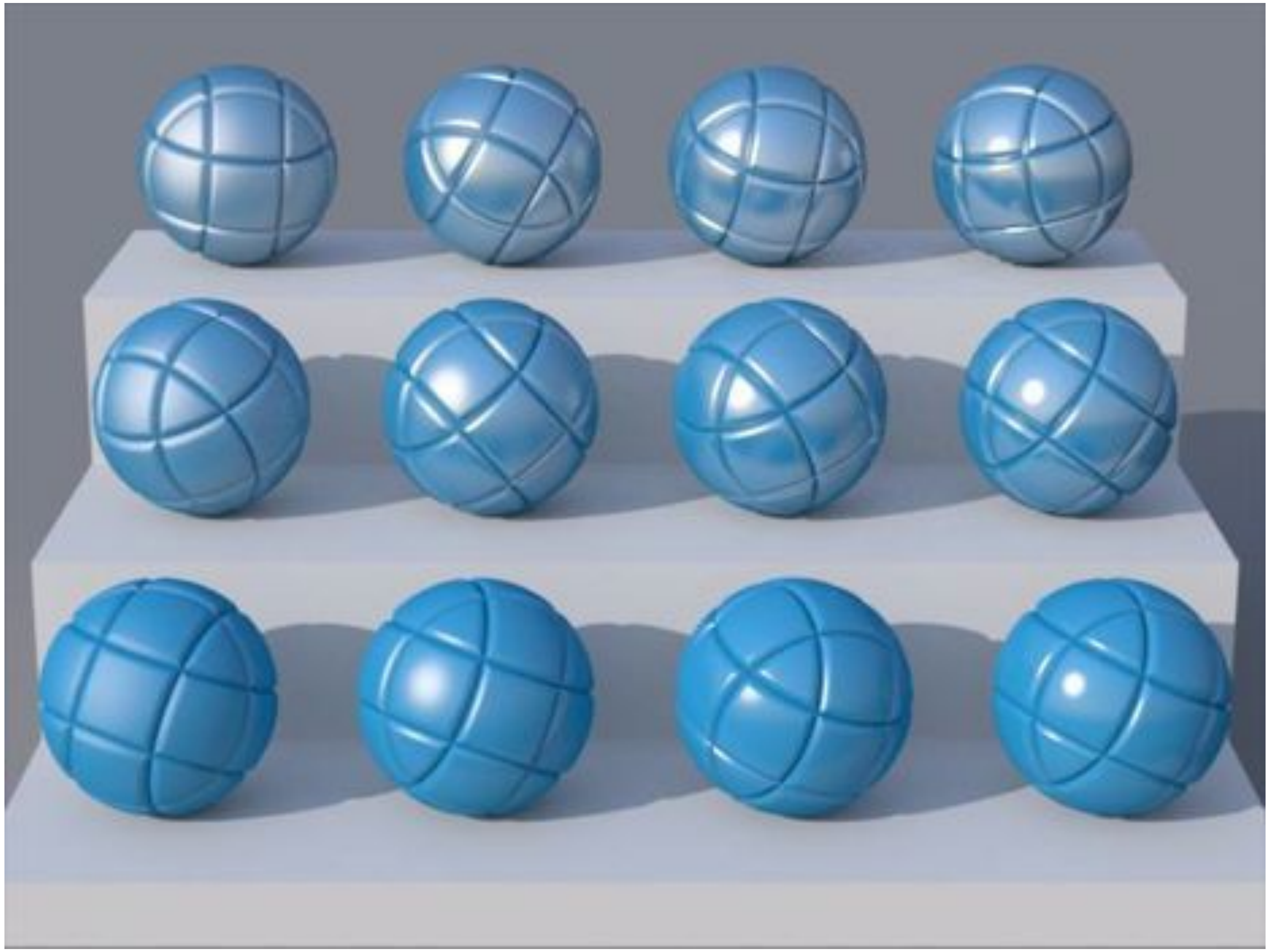
Cosine + Diffuse



Combined Phong Lobe

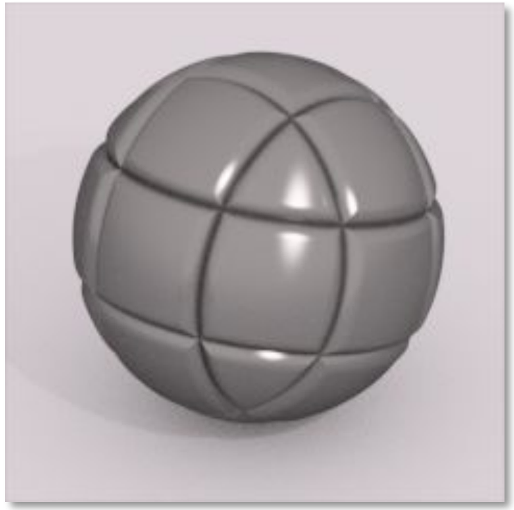


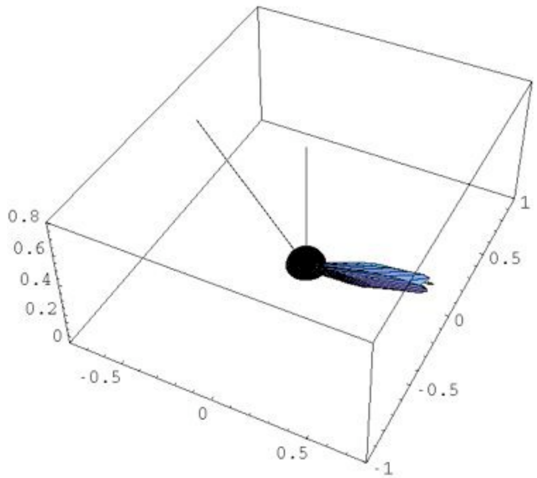
Phong Surface Example



Ward's BRDF Model

- Based on elliptical Gaussian distribution
- Physically based
- Energy conserving
- Based on real measurements with a gonio-reflectometer
- Analytically invertible



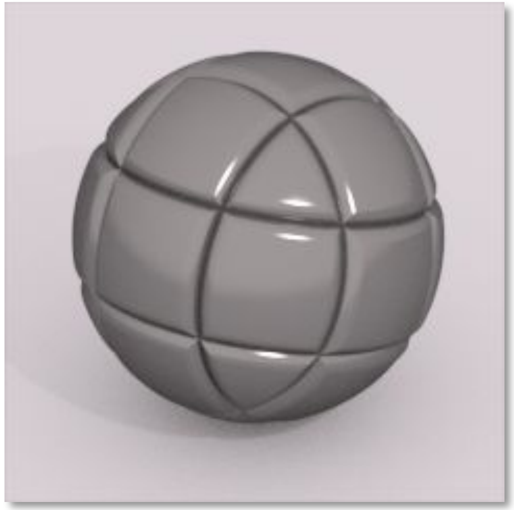


Ward BRDF Formulae

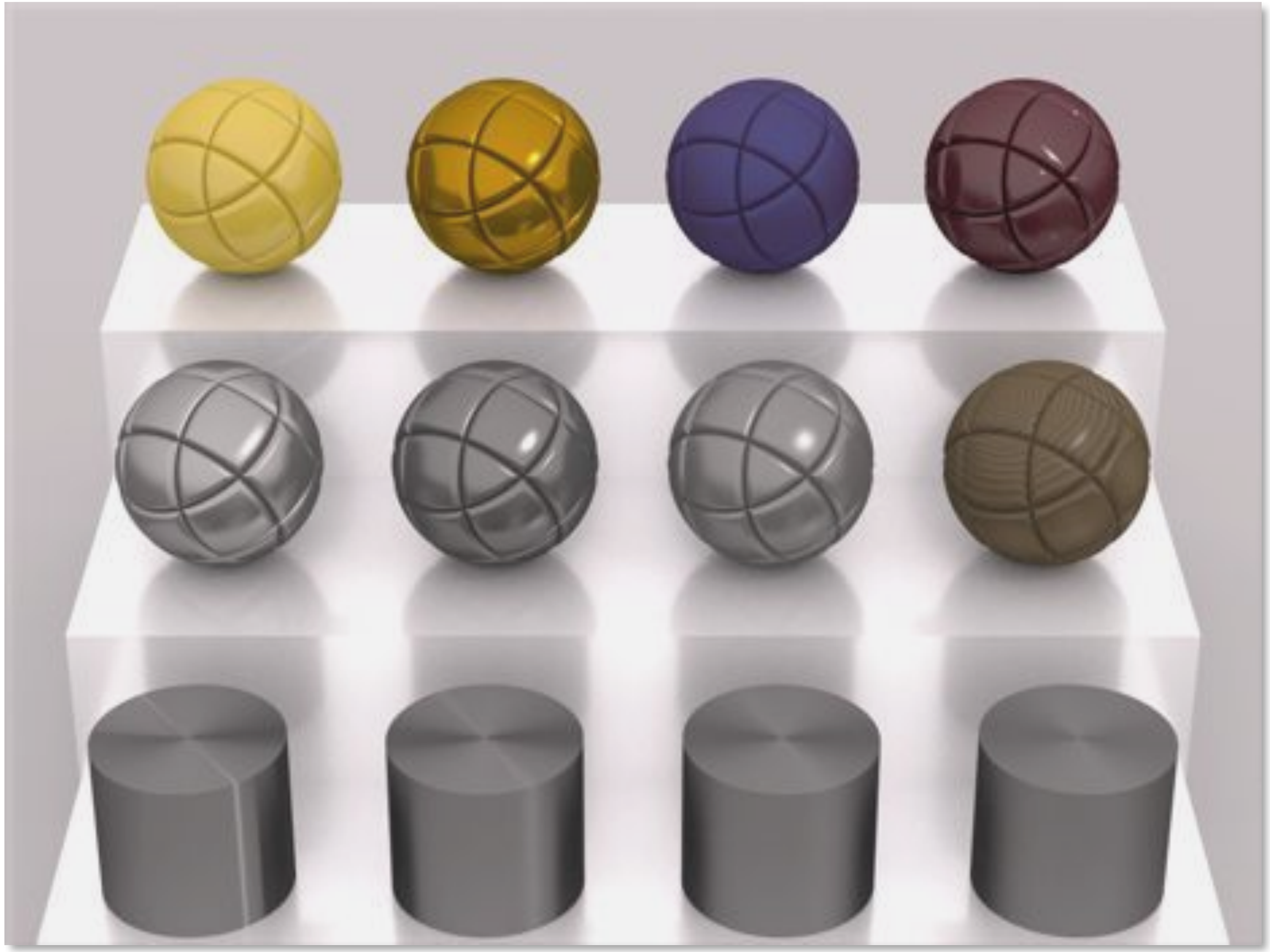
- Versions for isotropic and anisotropic surfaces

$$f_{iso}(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{\rho_d}{\pi} + \rho_s \frac{\exp(-\tan^2 \frac{\delta}{\alpha^2})}{4\pi\alpha^2\sqrt{\cos\theta_i \cos\theta_r}}$$

$$f_{an}(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{\rho_d}{\pi} + \rho_s \frac{\exp(-\tan^2 \delta(\frac{\cos^2 \phi}{\alpha_x^2} + \frac{\sin^2 \phi}{\alpha_y^2}))}{4\pi\alpha_x\alpha_y\sqrt{\cos\theta_i \cos\theta_r}}$$



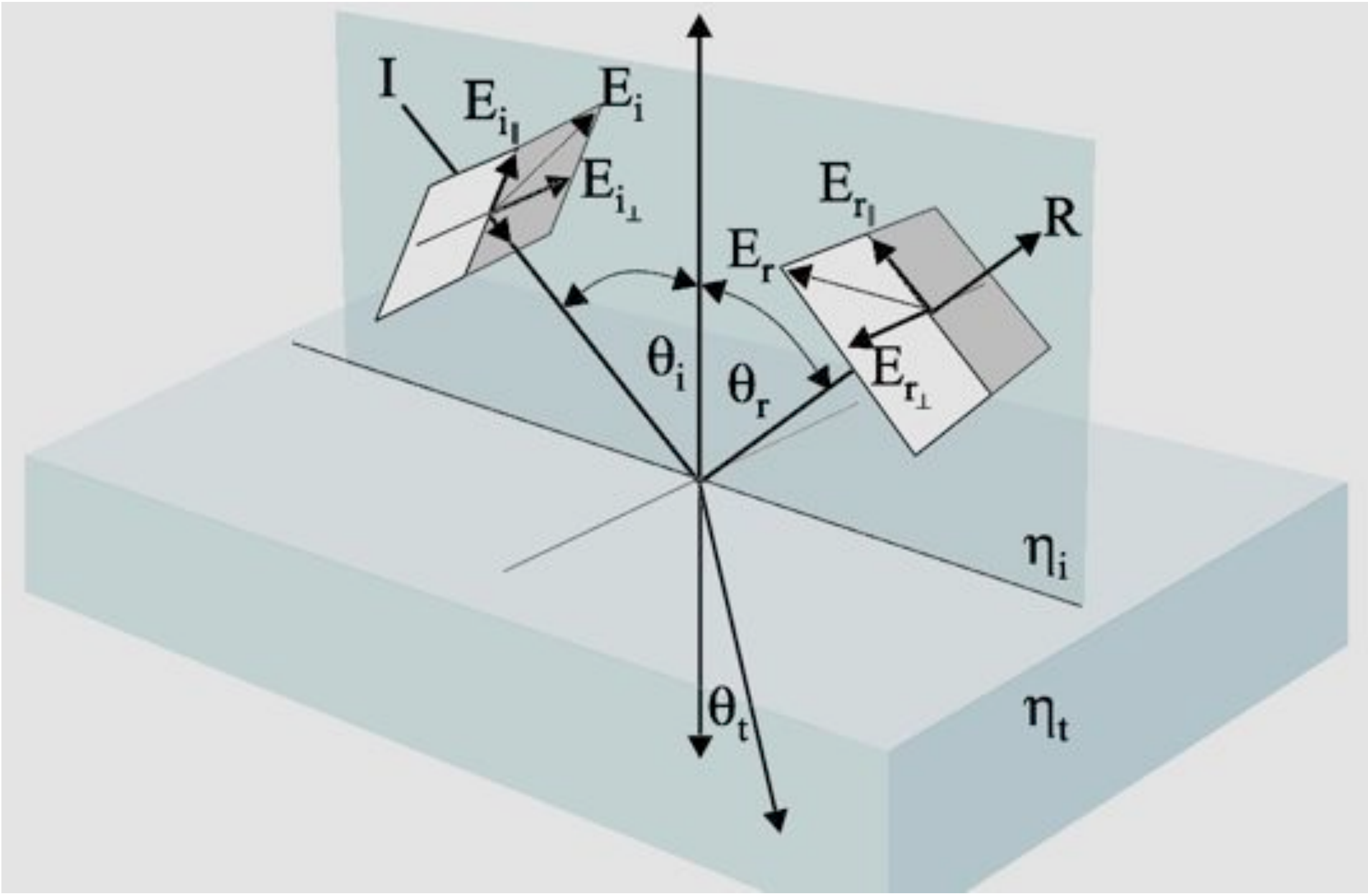
Wards BRDF: Example



Perfectly Specular

- Do not exist in reality
- Only one outgoing direction
- Incoming angle equals outgoing angle
- Used to simulate smooth glass / metallic surfaces
- For realistic materials: Fresnel coefficients

Fresnel Geometry



Fresnel Surface (Conductors)

- Interface between two materials with complex index of refraction
- Polarization information usually not used in renderer
- (n,κ): complex-valued index of refraction!

$$r_\perp = \frac{a^2 + b^2 - 2a\cos\theta_i + \cos^2\theta_i}{a^2 + b^2 + 2a\cos\theta_i + \cos^2\theta_i}$$

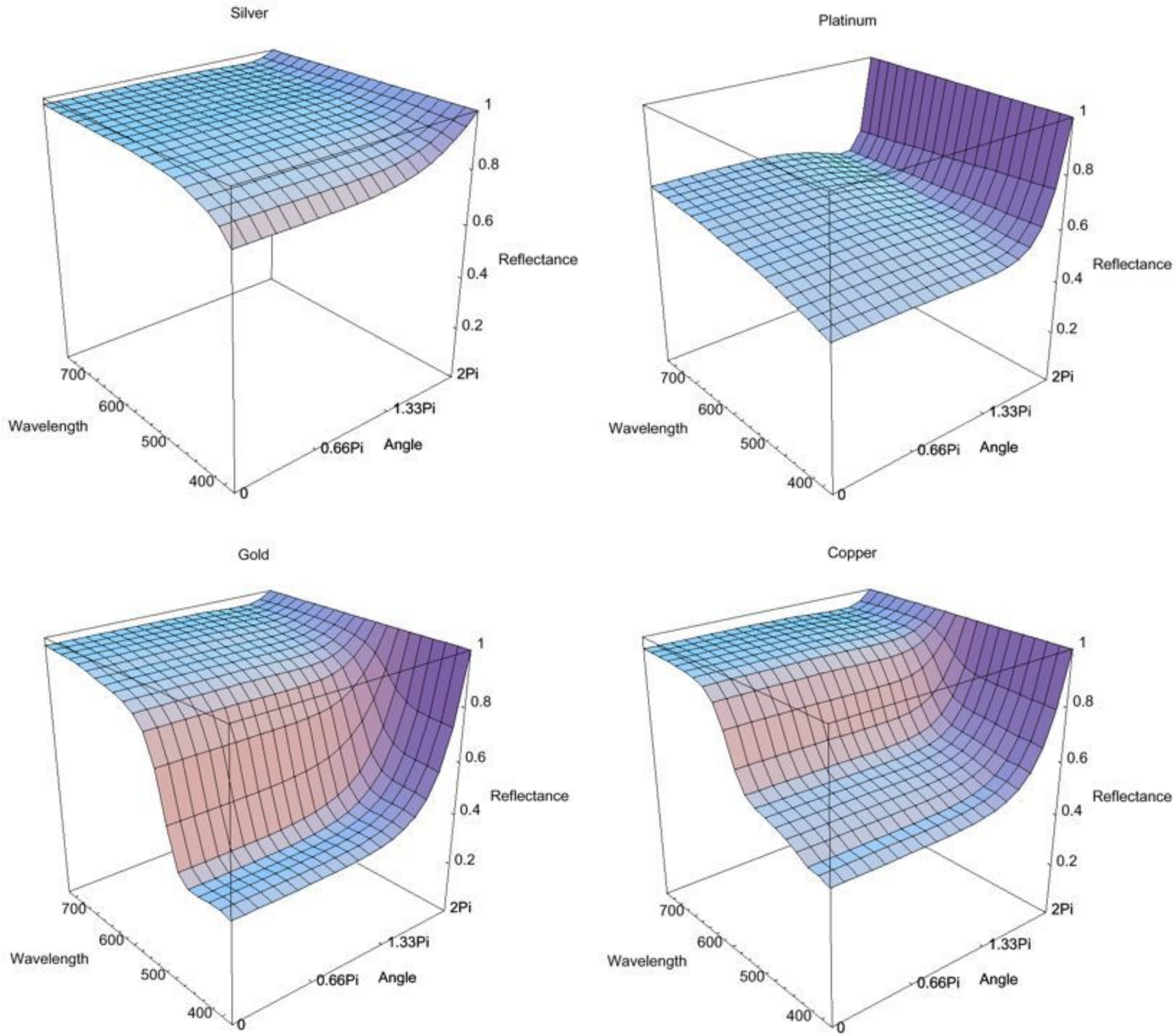
$$r_\parallel = r_\perp \frac{a^2 + b^2 - 2a\sin\theta_i\tan\theta_i + \sin^2\theta_i\tan^2\theta_i}{a^2 + b^2 + 2a\sin\theta_i\tan\theta_i + \sin^2\theta_i\tan^2\theta_i}$$

$$2a^2 = \sqrt{(\eta^2 - \kappa^2 - \sin^2\theta_i)^2 + 4\eta^2\kappa^2} + (\eta^2 - \kappa^2 - \sin^2\theta_i)$$

$$2b^2 = \sqrt{(\eta^2 - \kappa^2 - \sin^2\theta_i)^2 + 4\eta^2\kappa^2} - (\eta^2 - \kappa^2 - \sin^2\theta_i)$$

$$f_r(p, \omega_i, \omega_o) = F_r(\omega_o)\frac{\delta(\omega_i - R(\omega_o, \mathbf{n}))}{|\cos\theta_i|}$$

Conductors: Spectral Reflectivity



Reflection from Conductors

Fresnel Surface (Dielectrics)

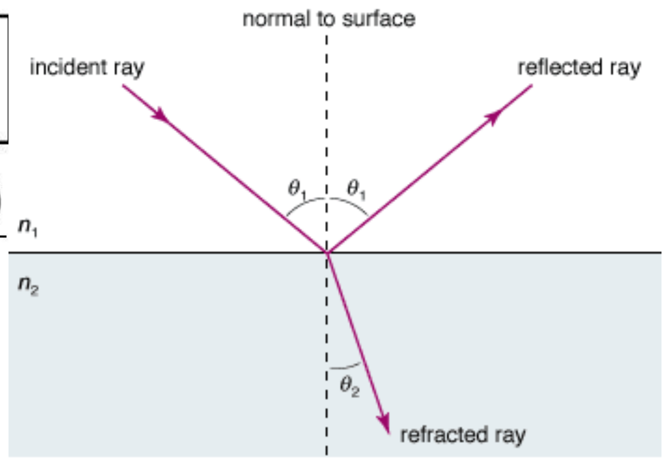
- Interface between two materials with real index of refraction
- No absorption k

$$r_{\parallel} = \frac{\eta_t \cos\theta_i + \eta_i \cos\theta_t}{\eta_t \cos\theta_i - \eta_i \cos\theta_t}$$

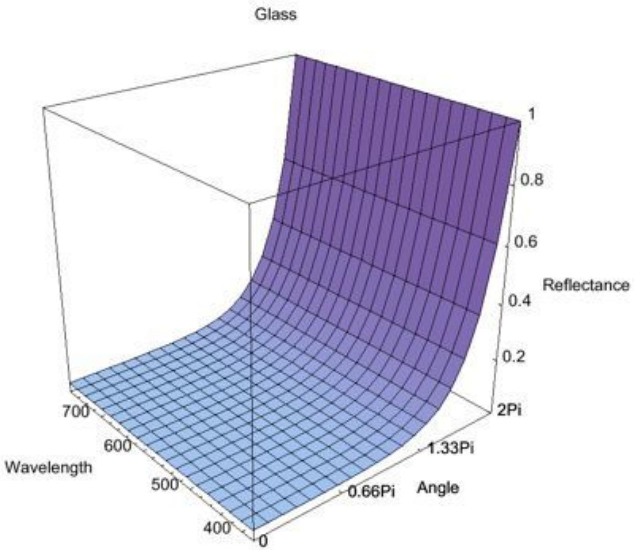
$$r_{\perp} = \frac{\eta_i \cos\theta_i + \eta_t \cos\theta_t}{\eta_i \cos\theta_i - \eta_t \cos\theta_t}$$

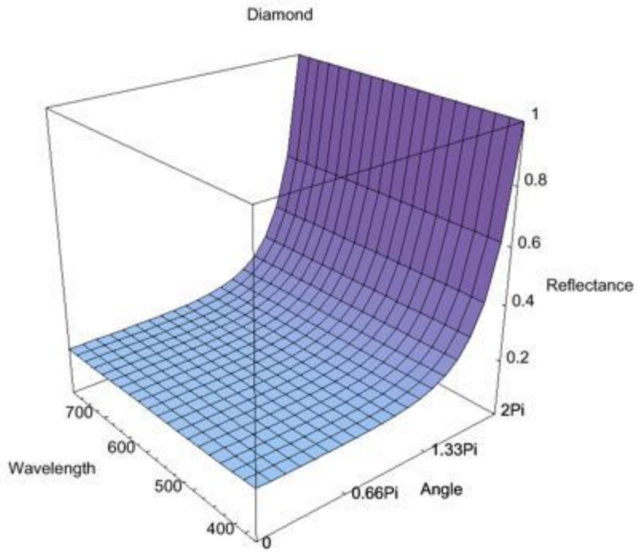
$$\mathrm{R} = \frac{1}{2}\left(\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}\right)^2 \left[1 + \left(\frac{\cos(\theta_i + \theta_t)}{\cos\theta_i - \theta_t}\right)^2\right]$$

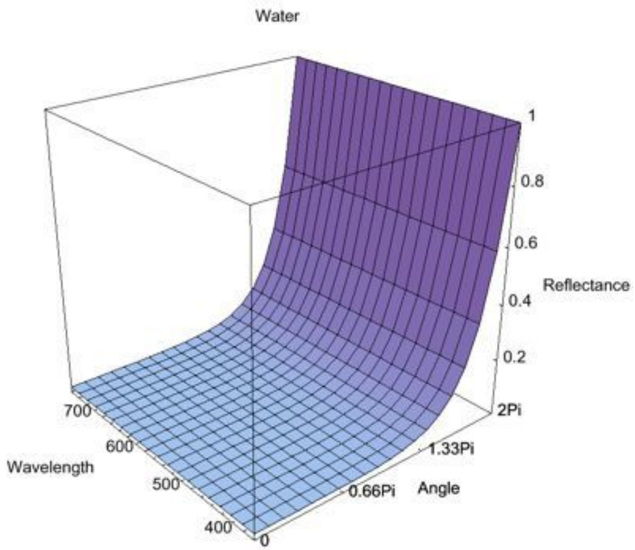
$$f_t(p, \omega_i, \omega_t) = \frac{n_o^2}{n_i^2}(1 - F_r(\omega_o))\frac{\delta(\omega_i - T(\omega_i, \mathbf{n}))}{|\cos\theta_i|}$$



Dielectrics: Spectral Reflectivity

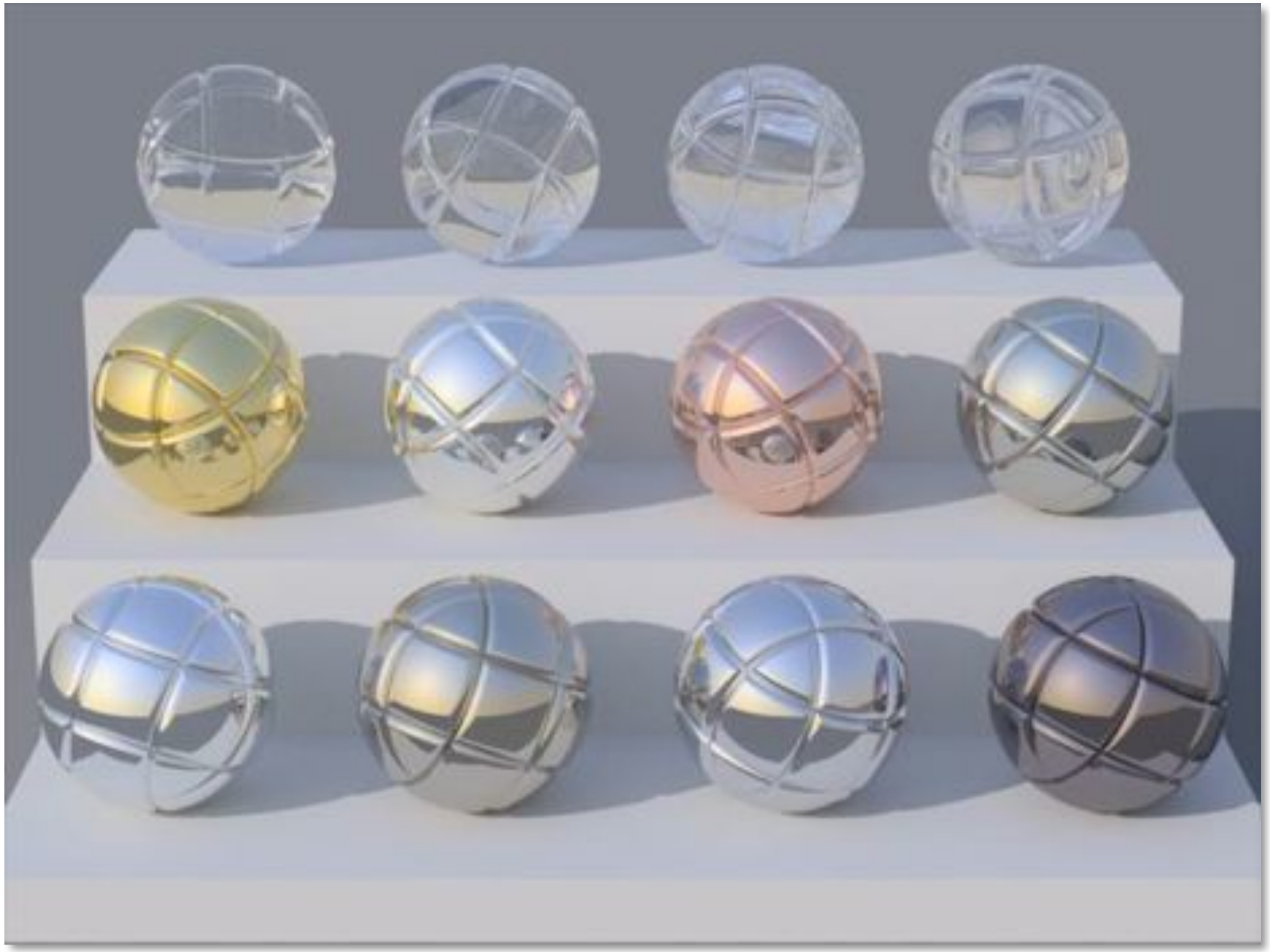






Transparent Dielectrics

Fresnel Surface Example



Directional Diffuse

- Combination of a rough specular reflector and an ideal diffuse reflector
- Eg.
  - Cook-Torrance
  - Ward
  - He
  - ...

Torrance-Sparrow Surface (1)

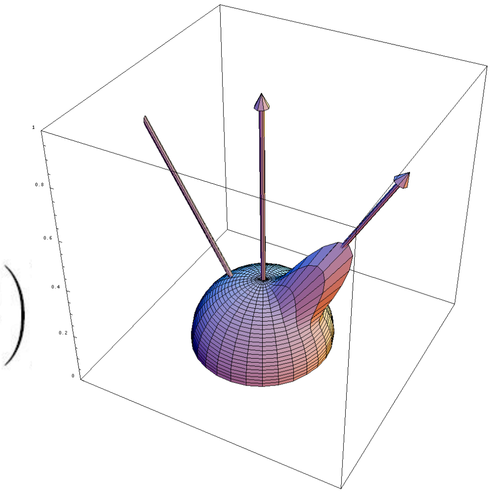
- Physically plausible BRDF model three main components:
  - Microfacet model
  - Fresnel term for reflectance
  - Roughness term
- Requires material constants to be known

$$f_r(p, \omega_i, \omega_o) = \frac{D(\omega_h)G(\omega_i, \omega_o)F(\omega_o)}{4\cos\theta_i \cos\theta_o}$$

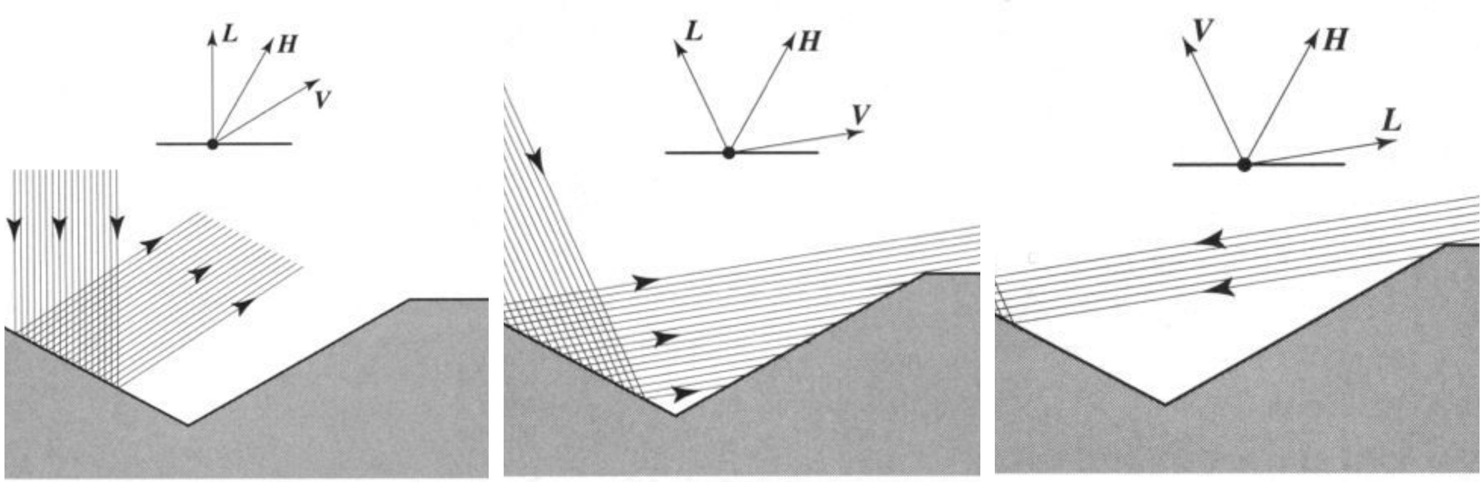
$$D(\omega_h) = \frac{e+2}{2\pi}(\omega_h \cdot \mathbf{n})^e$$

$$G(\omega_i, \omega_o) = \min\left(1, \min\left(\frac{2(\omega_h \cdot \mathbf{n})(\omega_o \cdot \mathbf{n})}{\omega_h \cdot \omega_o}, \frac{2(\omega_h \cdot \mathbf{n})(\omega_i \cdot \mathbf{n})}{\omega_h \cdot \omega_o}\right)\right)$$

$$pdf(\omega_h, \omega_i, \omega_o) = \frac{(n+2)\cos^n \theta_h}{4(\omega_h \cdot \omega_o)}$$



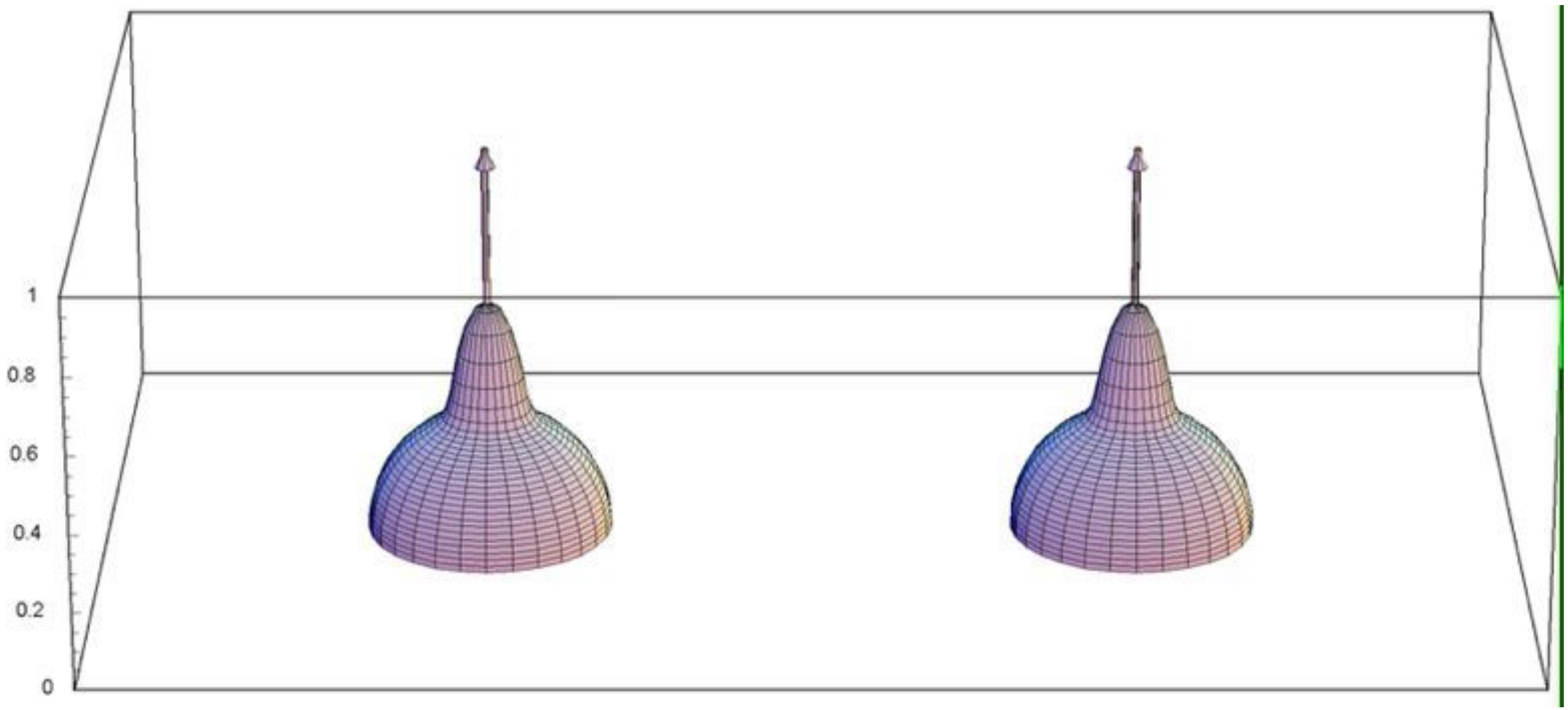
Geometric Factor



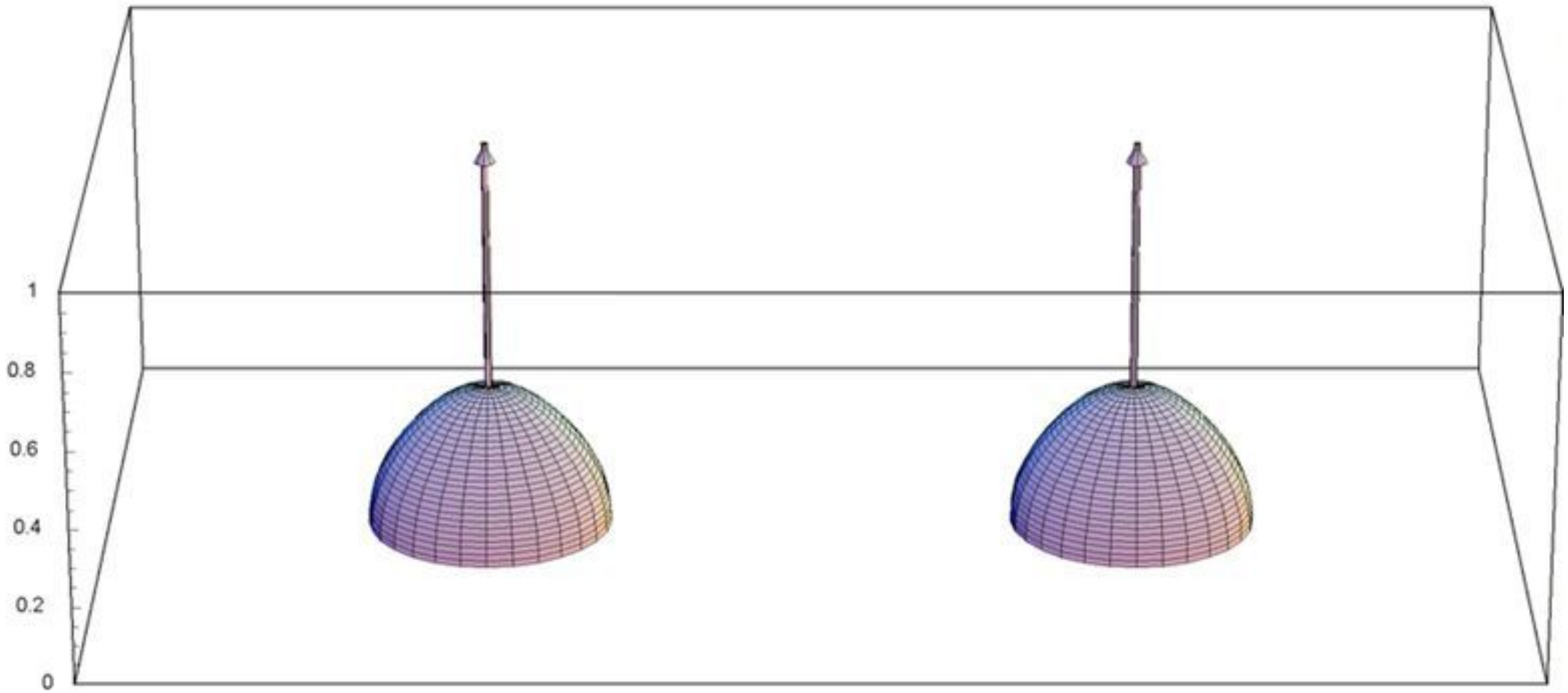
- Randomly oriented facets are perfect reflectors except for geometrical attenuation due to
  - self-shadowing
  - masking

$$\mathcal{G} = \min\left(1, \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{V})}{\mathbf{V}\cdot\mathbf{H}}, \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{L})}{\mathbf{V}\cdot\mathbf{H}}\right)$$

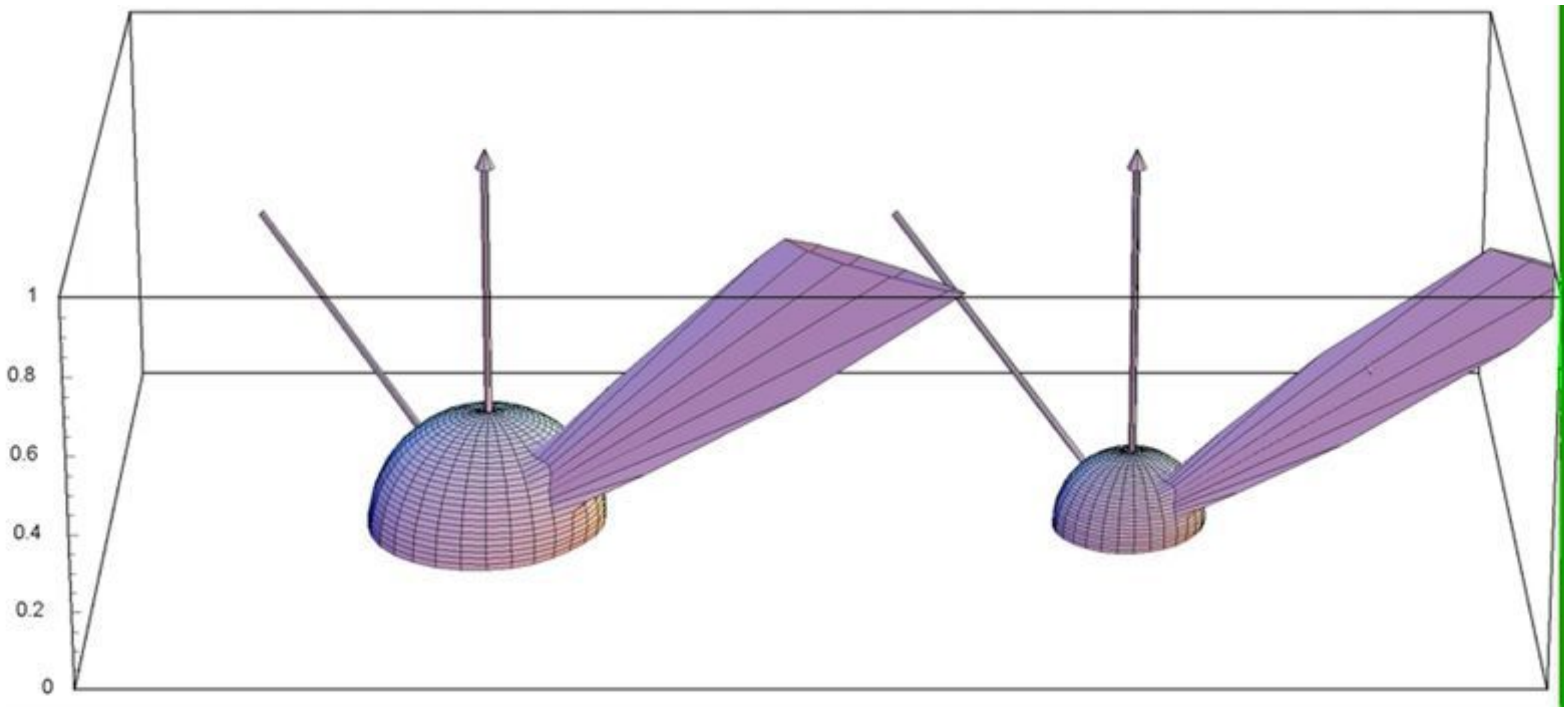
Specular Torrance-Sparrow



Rough Torrance-Sparrow



Constant Angle, Varying Roughness



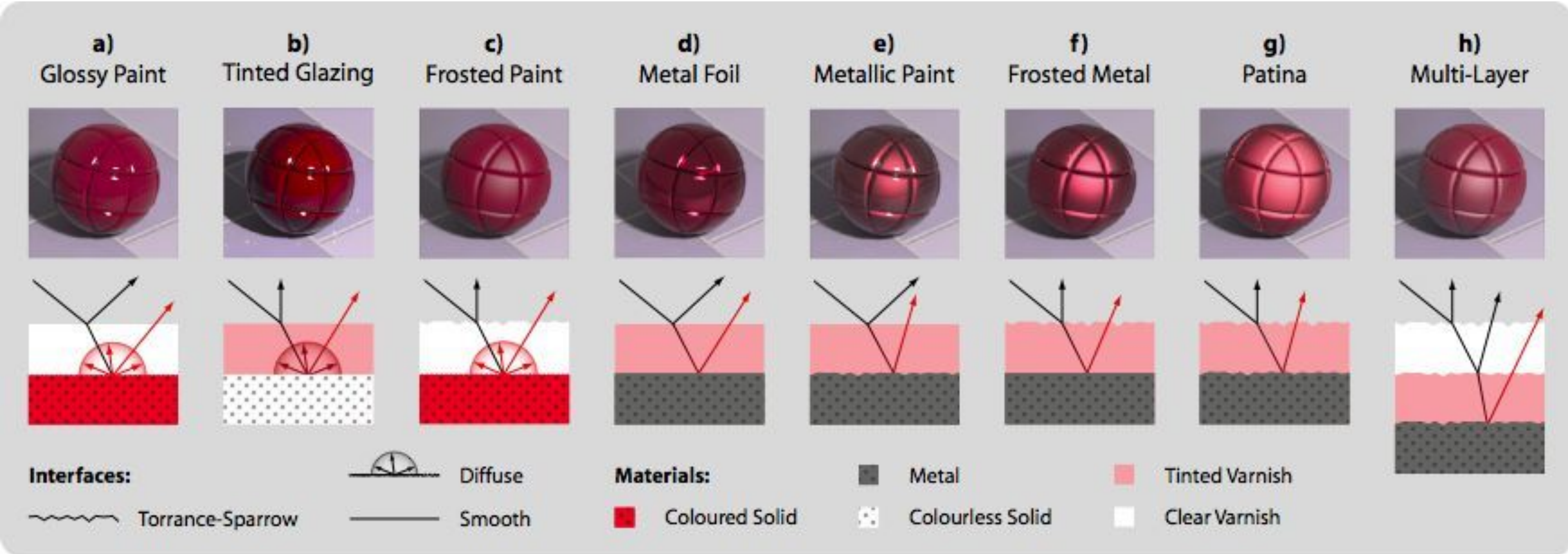
Rough Metallic/Glass Surface

Reflection Colour

Dielectrics: white highlight

Conductors: coloured highlight

Torrance-Sparrow Surface Types



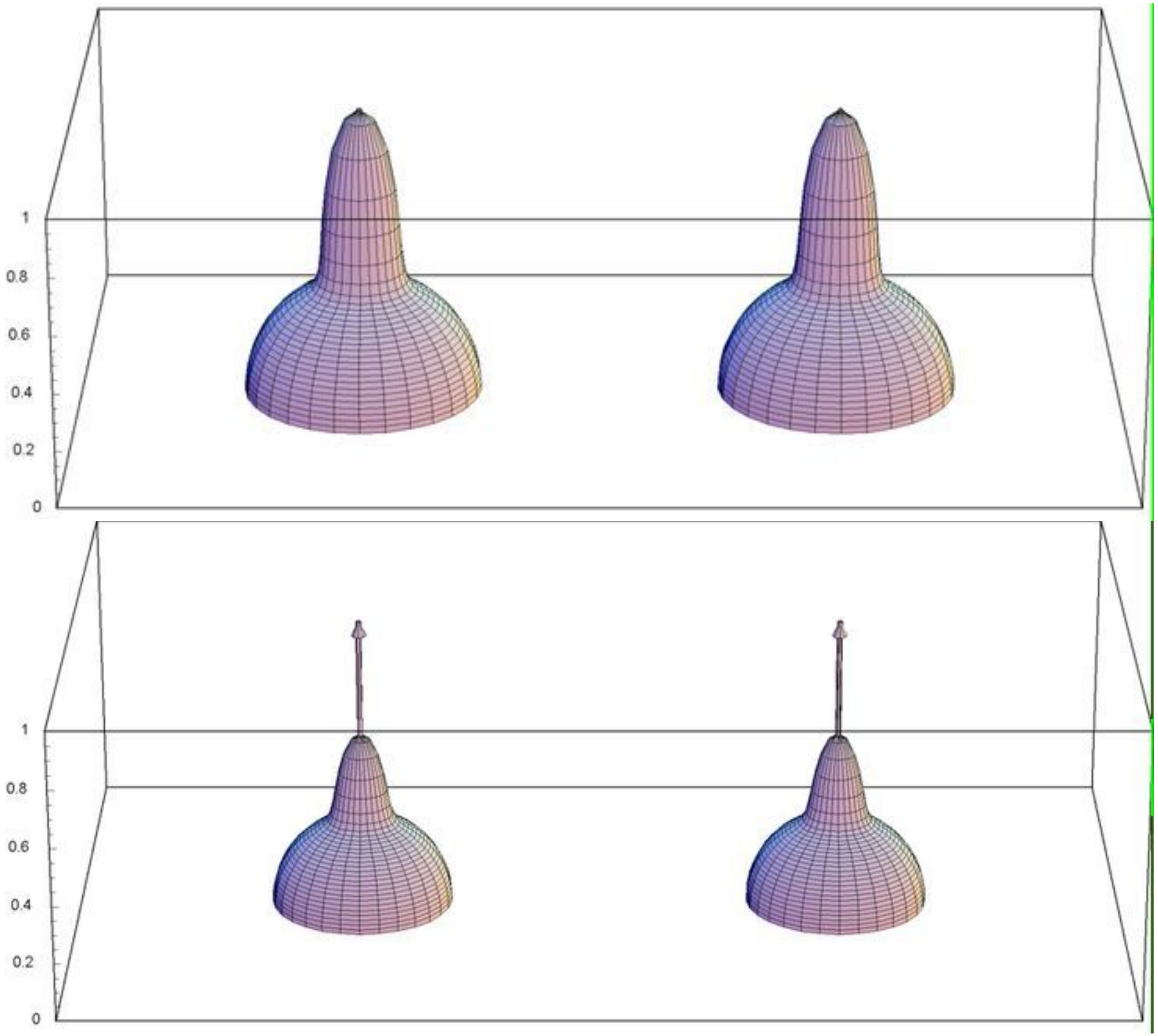
Layered Materials

Torrance-Sparrow vs. Phong

Torrance-Sparrow

Phong

Phong vs. Torrance-Sparrow

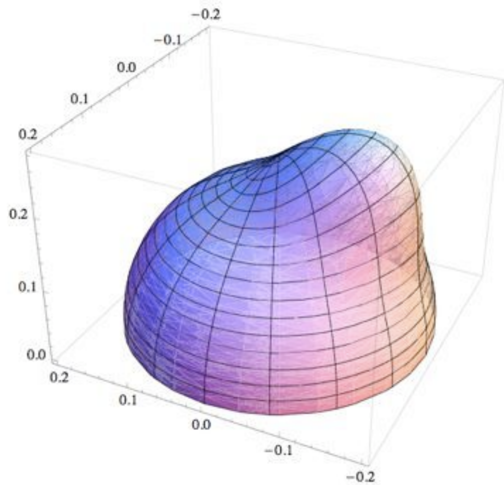


Torrance-Sparrow: Evaluation

- Plus:
  - physically correct
  - excellent results
- Minus:
  - hard to sample
  - hard to code
  - depends on material constants

He, Torrance, Sillion, Greenberg

- Based on wave optics and diffraction theory, can take polarization into account
- Additional split between diffuse and directional diffuse term
- Expensive to compute
- Input: auto-correlation τ, variance of surface height σ, IOR



He Implementation

- Specular component
  - Dirac impulse
- Directional diffuse component
  - Solve z_0 with Newton-Raphson
  - Infinity sum should have at least 250 terms → precomputation possible
  - Sampling: ???
- Uniform diffuse component
  - Cosine sampling

Beyond Normal BRDFs

- Some surfaces that cannot be characterized through standard BRDFs:

- Phosphorescent paint
- Fluorescent paint
- Metallic paint
- Pearlescent paint
- BSSRDFs

The End
Thank you for your attention!